Journal of the American Concrete Institute

Part 2, SEPTEMBER 1960

ACI PROCEEDINGS V. 56

THE FOLLOWING PAGES, 1385 - 1479 of Proceedings V. 56, contain discussion of JOURNAL papers published January through March, 1960, in accordance with the plan for quarterly publication of discussion.

Those readers who assemble their own Proceedings volumes for binding should plan to remove these pages to add to Proceedings V. 56, which will be completed with Part 2, December 1960 JOURNAL, which contains contents, indexes, errata, and final quarterly installment of discussion.

Contents of the following pages are listed on the outside back cover of this JOURNAL.

Discussion of a paper by William A. Cordon:

Evaluation of Concrete and Mortar Mixes*

By S. A. MARKESTAD†

Generally speaking, I don't imagine that the study of concrete by studying the properties of its released mortar will gain much popularity for practical work. This is due to reasons which the author has put forward (see the bottom of p. 570).

The opposite problem: evaluation of sands based on the mortar test with regard to their suitability in concrete is most interesting. For this evaluation the results given by the author and the author's method of elucidating the relationships in mortars (Fig. 2) are of great concern. As to the cone penetrometer described by the author for measuring consistency and workability of concrete mortars, we have also found that it gives very reproducible results. At the same time this method is convenient and rapid.

My comments are concerned with the section "Grading of sand" and specially what is said regarding Fig. 2: "By moving horizontally the various sands may be evaluated for strength and water requirement at the same workability."

This method corresponds to the Norwegian standard method as used for 30 years: a constant cement content (constant cement-sand ratio) and water to a given consistency. This method results in greater compressive strength for very coarse sand, lower strength for medium graded sand, and the lowest strength for very fine sand in accordance with the water requirements of the sand. Fig. 2 shows this relationship and not, as stated by the author, that the medium graded sand shows the greater strength.

In evaluating sands based on the mortar test the heavy influence of the fineness moduli thus have to be taken into consideration. Particularly, one ought to remember that the finer the sand to be used in concrete the smaller the necessary percentage of sand. Thus the cement-sand ratio will be higher in the concrete by using a fine sand. This may be attained without raising the cement content in kg per cu m of concrete. This richer mortar will compensate for the higher water re-

*ACI Journal, V. 31, No. 7, Jan. 1960 (*Proceedings* V. 56), p. 569. Disc. 56-34 is a part of copyrighted Journal of the American Concrete Institute, V. 32, No. 3, Part 2, Sept. 1960 (*Proceedings* V. 56).

†Member American Concrete Institute, Professor and Head, Materials Testing Department, Norges Tekniske Hogskole, Trondheim, Norway.

quirement of the finer sand. Properly proportioned concretes containing different graded sands give approximately the same compressive strengths at a given water-cement ratio. Thus a comparison of sands should preferably be performed *by mortars of given water-cement ratios and at the same time at a given consistency.* In Norway this is the direction of change regarding the mortar test method. By this revision the evaluation of results obtained by the mortar test will be much easier than before.

The test could then be carried out by making mortars of say $w/c = 0.50 \pm 0.5$; 0.70 ± 0.05; and 1.00 ± 0.05 thus changing the cement-sand ratio for each case according to the fineness moduli and other properties which govern the water requirement. By graphical interpolation those strengths will be given which correspond to $w/c = 0.50$, 0.70, and 1.00.

Returning to Fig. 2 the author says further: "By moving vertically the same data can be used to evaluate workability and *strength* for the same water-cement ratio."

I agree that Fig. 2 is excellent for evaluation of workability. But regarding the strength for the same *water-cement ratio* it is *not* applicable as the mortars compared do not have the same consistency. It is of course unfair against the coarse sand, F.M. 3.02, to compare the strength results obtained from a mortar of consistency as high as 80 with a mortar of consistency as low as 40 for a finer sand, F. M. 2.33.

In evaluating sands based on the strengths obtained thus, only the first method, by moving horizontally, is applicable. This method gives results which are somewhat difficult to correlate with the properties of sands when used in concrete. These difficulties may be overcome by the method indicated above using a constant water-cement ratio and constant consistency as a basis for comparison.

Discussion of a paper by Henry J. Cowan:

Design of Beams Subject to Torsion Related to the New Australian Code*

By BORIS BRESLER, BRUCE H. FALCONER, HANS GESUND, THOMAS PAULAY, and PAUL ZIA

By BORIS BRESLER†

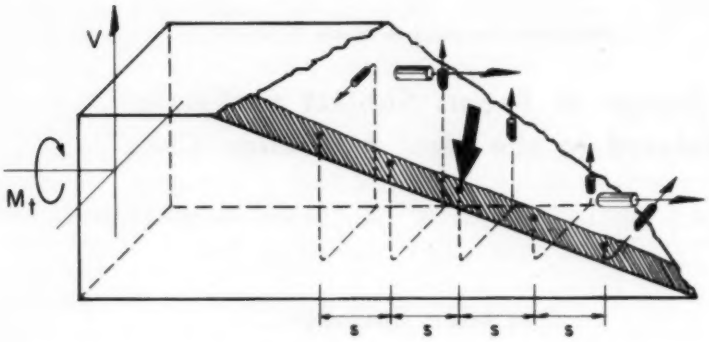
Professor Cowan's paper on design of reinforced concrete beams subjected to torsion is a valuable survey of the information available on this subject. In order to complete this extensive survey, the studies of torsion in reinforced concrete beams carried out in USSR and reported in recent publications[27-30] are briefly reviewed below. These studies included analytical and experimental investigations of strength of reinforced concrete rectangular beams subjected to combined torsion, bending, and transverse shear, and were carried out by Miss N. N. Lessig and her colleagues under the direction of Prof. A. A. Gvozdev, during 1948-1956.

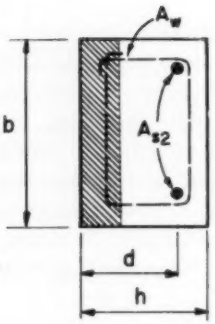
An idealized collapse mechanism was formulated based on development of "spiral" cracks along three sides of a rectangular beam, followed by yielding of the steel, and by rotation of two adjacent segments of the beam with respect to each other, forming a plastic hinge. The hinge rotation about a skew axis has a flexural and a twisting component. Final failure is due to crushing of the concrete after yielding of both the longitudinal and the transverse reinforcement. In order to assure initiation of failure by yielding of the reinforcement, the amount of longitudinal and transverse steel must be within prescribed limits, similar to the balanced reinforcement limit for yielding failure in flexure.

Two particular failure conditions were studied: in one case the shear force was relatively small and failure occurred due to combined effects of bending and torsion; in the other case the bending moment was relatively small and failure occurred due to combined effects of shear and torsion. It was shown that when shear force was negligible failure occurred by rotation about an axis in a horizontal plane normal to the plane of bending moments, Fig. A. When the bending moment was

*ACI JOURNAL, V. 31, No. 7, Jan. 1960 (*Proceedings* V. 56), p. 591. Disc. 56-36 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 32, No. 3, Part 2, Sept. 1960 (*Proceedings* V. 56).

†Member American Concrete Institute, Professor of Civil Engineering, University of California, Berkeley, Calif.





$$\phi = M_t / Vh$$

$$a = b/h$$

$$p_2 = A_{s2} / bd$$

$$v = V / bd$$

$$r = 1 + 2/a$$

$$\alpha = 1 + \frac{f_{yw} A_w}{f_y A_{s2}} \frac{b}{s} (1 + 2/a)$$

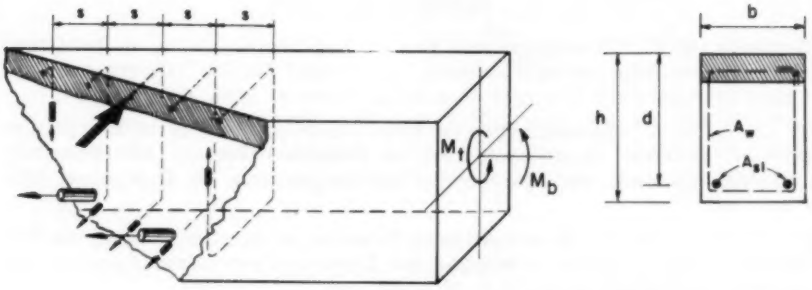
$$\lambda_2 = \frac{a - v/p_2 r f_y}{2(1 + r^2)}$$

$$K_v = \frac{a}{r(1 - 2/\phi)}$$

Fig. A

negligible, failure occurred by rotation about an axis which can be closely approximated by one in a vertical plane parallel to the shearing force, Fig. B.

Assuming yielding in all the reinforcing bars and crushing of all the concrete in the compression zone, equations defining ultimate bending moments, shear, and torsional moments were derived using equations of equilibrium or by equating external and internal work. These equations were derived on the basis of the following assumptions: (1) all reinforcement yields before crushing occurs, (2) tensile strength of concrete is neglected, (3) transverse reinforcement is uniformly distributed throughout the beam, (4) the zone of failure is free from local applied loads, and (5) plane of shear force V and of bending moment M_b is parallel to the vertical edge of the rectangular beam. The position of the hinge axis was determined by minimizing values of moment or shear in above equations with respect to the depth of compression zone.



$$\phi = M_t / M_b \qquad \alpha = \frac{f_{yw} A_w}{f_y A_{sl}} \frac{b}{s} \frac{r^2}{(1 + 2/\alpha)}$$

$$\alpha = b / h \qquad \lambda_1 = \frac{1 + \alpha}{2(1 + r^2)}$$

$$p_l = A_{sl} / bd \qquad K_b = \phi \left(\frac{1 + \alpha}{1 + r\phi} \right)$$

$$r = \sqrt{\left(\frac{1}{\phi}\right)^2 + \frac{f_y A_{sl}}{f_{yw} A_w} \frac{s}{b} \left(1 + \frac{2}{\alpha}\right)} - \frac{1}{\phi} \leqslant \left(1 + \frac{2}{\alpha}\right)$$

Fig. B

Simplifying the general equations for practical cases the following approximate expressions were developed for the two cases studied. For the case of combined bending moment M_b and torsional moment M_t, and using notation shown in Fig. A

$$\frac{M_t}{bd^2 f_c} = K_b \left(p_1 \frac{f_y}{f_c} \right) \left(1 - \lambda_1 p_1 \frac{f_y}{f_c} \right)$$

where

f_y, f_{yw} = yield point stress of longitudinal and transverse steel reinforcement, respectively

f_c = crushing strength of concrete prism, approximately equal to $0.85 f_c'$

For the case of combined shear V and torsional moment M_t, and using notation shown in Fig. B (note: b and h are reversed)

$$\frac{M_t}{bd^2 f_c} = K_v \left(p_2 \frac{f_y}{f_c} \right) \left(1 - \lambda_2 p_2 \frac{f_y}{f_c} \right)$$

For given beam geometry and given values of ϕ the ultimate torsional moment M_t is easily determined from the above relationships.

Experimental investigations reported in above references describe five series of tests, including 72 rectangular beam specimens. A number of different variables have been investigated, and a fairly good agreement between experimental results and proposed analytical relationships has been demonstrated.

REFERENCES

27. Lessig, N. N., "Theoretical and Experimental Investigation of Reinforced Concrete Beams Subjected to Combined Bending and Torsion," *Design and Construction of Reinforced Concrete Structures*, Moscow, 1958, (in Russian).

28. Lessig, N. N., "Determination of Load Carrying Capacity of Rectangular Reinforced Concrete Beams Subjected to Combined Torsion and Bending," *Proceedings*, Concrete and Reinforced Concrete Institute, V. 5, Moscow, 1959 (in Russian).

29. Chinenkov, U. V., "Investigation of Behavior of Reinforced Concrete Elements Subjected to Combined Bending and Torsion," *Proceedings*, Concrete and Reinforced Concrete Institute, V. 5, Moscow, 1959 (in Russian).

30. Lialin, I. M., "Experimental Investigation of Behavior of Rectangular Reinforced Concrete Beams Subjected to Combined Shear and Torsion," *Proceedings*, Concrete and Reinforced Concrete Institute, V. 5, Moscow, 1959 (in Russian).

By BRUCE H. FALCONER*

Shear stresses

For approximate assessments of shear stress in solid sections under torsion, the discusser considers a peripheral band of a cross section as if it is the wall of a hollow box, and computes the equivalent Bredt-Batho shear stress. For a rectangular section, using a band width equal to 1/5 of the smaller side, the torsional moment is,

$$M_t = \text{twice area of box by the shear flow}$$

$$M_t = 2\left[\frac{4x}{5}\left(y - \frac{x}{5}\right)\right]\frac{x}{5}\,v_t = \frac{8}{25}\,x^2\left(y - \frac{x}{5}\right)v_t \qquad (20)$$

or

$$M_t = \alpha\, x^2\, y\, v_t, \text{ where } \alpha = 0.32\,\frac{y/x - 1/5}{y/x} \qquad (21)$$

A comparison of the parameters α given by the author, the Australian Code, and the discusser, is given in Fig. C.

Correspondingly, an assessment of the required reinforcement is made for the shear flow, $M_t/2x_1y_1$, according to customary procedure for the reinforcement of concrete for transverse shear.

Should it be desired to provide reinforcement able to carry the full tensions due to shear the required stirrup area is

$$A_v = 2\psi\,\frac{M_t s}{2x_1y_1f_s} = \psi\,\frac{M_t s}{x_1y_1f_s} \qquad (22)$$

and the required sum of areas of longitudinal bars is

*Member American Concrete Institute, Consulting Engineer, Wellington, New Zealand.

$$A_{s1} = \frac{1}{\psi} \frac{M_t}{f_s} \frac{x_1 + y_1}{x_1 y_1} \quad \text{.....(23)}$$

$$A_{s1} = \frac{1}{\psi^2} A_{sv} \frac{x_1 + y_1}{s} \quad \text{.....(24)}$$

where ψ is generally taken as unity, but may be chosen arbitrarily within limits, say in the range $2/3 < \psi < 1$. For $\psi < 1$ it must be assumed that the angle of compression in the concrete is less than 45 deg to the axis of the beam.

This simple approach ignores the variation in stress at the surfaces of the longer and shorter sides, because the author's parameter λ is taken as unity. Otherwise there is agreement with the author's Eq. (17) and (19).

Surprise is felt that the Australian Code allows torsional shear stress (and combined torsional and transverse shear stress) up to 360 psi, with hoop stirrups carrying only the excess over $0.02 f_c' + 20$ psi (90 psi maximum). This is so particularly in view of the requirement of Section 614 (c) and (e) that for 180 psi $< v <$ 360 psi transverse shear the whole of the shear must be carried by shear reinforcement, using two types of shear reinforcement, with each type being credited with not more than half the total shear resistance.

Greater surprise, however, is felt that the longitudinal reinforcement need be proportioned for only the excess shear. In view of the general likelihood of construction joints (normal to the beam axis) and possible stress due to shrinkage restraints, would it not be preferable to provide, at least at a minimum, sufficient longitudinal steel for the full torque?

Torsional stiffness

Not infrequently theoretical analyses of the torsional stresses in beams of a building, made using a torsional rigidity equal to that of the uncracked (or lightly loaded) concrete section, indicate seriously high stresses, yet the designer has not considered the torsion, and the beams show no apparent distress.

It would seem that there are two aspects of the problem which warrant attention; (a) reinforcement to provide a given strength, and (b) reinforcement to sustain a given deformation.

These aspects, the latter particularly, primarily require experimental investigation. Nevertheless the discusser offers the following theoretical comment which may be relevant.

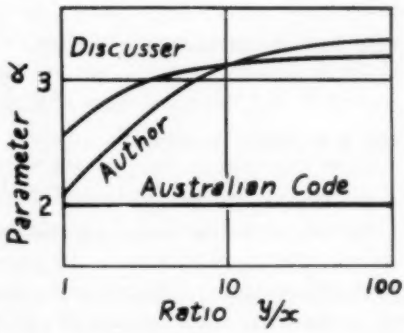


Fig. C

Consider, once more, the assumed analogous hollow box, subject to a twist of θ radians per unit length. One can relate the angle of twist of the box and the shear strain in the walling as follows,

$$\begin{matrix}\text{geometrical "slip" around} \\ \text{cross section}\end{matrix} = \begin{matrix}\text{shear distortion around} \\ \text{cross section}\end{matrix}$$

or

$$2\left(\theta\frac{x}{2}y + \theta\frac{y}{2}x\right) = 2(x+y)\chi$$

or

$$\theta = \frac{x+y}{xy}\chi \qquad (25)$$

where χ is the unit shear strain in the wall material, and is for an uncracked member

$$\chi = \frac{v}{G_c} = \frac{2(1+\mu)v}{E_c}$$

or for a cracked member

$$\chi = \frac{4v}{E_c} + \frac{2f_s}{E_s}$$

Assuming for example, $f_s = 20{,}000$ psi, $E_s = 30 \times 10^6$, $E_c = 3 \times 10^6$, $0 < v < 360$ psi, then

$$\frac{2(1+\mu)v}{E_c} < 0.30 \times 10^{-3}$$

$$\frac{4v}{E_c} < 0.48 \times 10^{-3}$$

$$\frac{2f_s}{E_s} = 1.33 \times 10^{-3}$$

and for an uncracked member,

$$\chi = 0.30 \times 10^{-3}$$

for a cracked member

$$\chi = 1.81 \times 10^{-3}$$

In this particular example the torsional rigidity of the cracked member is only $100 \times 0.30/1.81 = 16$ percent of that which would be conventionally assumed. The theory presumes uniform and frequent cracking, or stress-less tensile strain of concrete. To the extent that this assumption is in error, the actual reduction of torsional rigidity will be less severe.

By HANS GESUND*

The writer is grateful to Professor Cowan for taking the trouble to provide American engineers with a portion of the Australian Code whose counterpart is unfortunately lacking in our own Codes. At the writer's suggestion, a local consulting firm is already using the Australian formulas for the design of a girder in a coal tipple which has to resist great twisting moments.

At the same time, it is unfortunate that Professor Cowan and the writers of the new Australian Code described the action of reinforced concrete beams in torsion in terms of the elastic theories. The basic assumption there is that the steel works together with the concrete in tension, each taking part of the tensile force. This reasoning is, however, only applied to tension due to shear and has fortunately never been used in bending. Thus the total twisting moment is assumed to be equal to that which the uncracked concrete cross section will resist *plus* that which the steel will carry, stressed to an allowable stress which Professor Cowan sets at 18,000 psi in his examples.

The trouble is, that if the steel is to have that much stress in it, the concrete near it and supposedly working with it must surely be cracked, as is always taken for granted in bending theory. The *possibly* uncracked core will have relatively small dimensions and hence cannot contribute much torque resistance. Therefore, one of two things must be true in the torsional analysis. Either the steel stress must remain very low, in which case the type of action envisioned by the Code will actually be present, or else the concrete will crack and the transverse steel will take almost the entire twisting moment. The latter case was investigated by Prof. G. C. Ernst and reported in the author's Reference 13, whose date, incidentally, was October 1957, not 1951 as given in the list of references. By limiting the "diagonal tensile stress" f_t to $0.08\ f_c' + 80$ psi ≤ 360 psi, the Australian Code seems to be trying to ensure that the assumed action will actually take place. However, that stress is very near the limit of what the concrete can actually resist and thus it will sometimes crack. After the cracking the steel will have to resist almost all the torque, which is more than it was designed for. Furthermore, some cracks will already be present if there is also bending moment on the section. It is also important to note that if the section is once overloaded for any reason, as is possible during construction, the concrete will be cracked and the steel will henceforth have to carry the entire torque.

One question occurring to the writer in connection with combining bending moment and torque, is what the effect on the assumed crack

*Member American Concrete Institute, Associate Professor of Structural Engineering, University of Kentucky Lexington, Ky.

pattern will be. It is to be doubted that the cracks will still be inclined at 45 deg, since the principal stresses will be at another angle.

In spite of the above criticisms, which, of course, can also be, and have been leveled at the ACI Code provisions on shear reinforcement for beams,* the writer is thankful to have this code by which to guide his students. However, while members designed under it will probably be very similar to those designed by any other method, he would have preferred design formulas based on the failure concept of the action of the materials or at least of the concrete, as in bending theory. It does seem illogical to design for bending on the basis that the concrete takes no tension and then design for shear and torsion on the basis that it does.

By THOMAS PAULAY†

The writer is indebted to Professor Cowan for the valuable information presented in his paper and for the very useful bibliography. The new Australian Code appears to be the first attempt, amongst English speaking countries, to give a comprehensive guidance in the design of concrete beams subject to torsion.

The writer used similar equations which compare well with those recommended by the Australian Code. If it is assumed that the hoop reinforcement reaches the yield stress it may be assumed that there is a uniform stress around the hoop. By similarity to shear flow in a thin tube the stress in the hoop will be

$$f_s = \frac{M_t S}{2a_{st} x_1 y_1}$$

If the average working stress is limited to $0.8f_s$, Eq. (18) may be obtained.

It was shown that equal amount of vertical and horizontal reinforcement is required to resist effectively diagonal tension resulting from torsion. This requirement applies to transverse shear as well, as was shown by Rausch,[31] but is often overlooked.

The Code requires at least one bar in each of the four corners of the beam as horizontal torsion reinforcement. It is felt that this concession should be used for small beams only. For the vertical spacing of horizontal bars in the web of a beam the writer used the lesser of the following dimensions:

$$x_1 \text{ or } 0.5y_1 \text{ or } 12 \text{ in.}$$

As torsional shear cancels transverse shear in one half of the cross section, when a beam is subject to both, it appears that the permissible maximum combined shear stress could be slightly increased.[32]

---

*Discussion 55-45, ACI JOURNAL, V. 30, No. 12, June 1959 (*Proceedings* V. 55), p. 1432.

†Member American Concrete Institute, D. Bruce-Smith and Associates, Consulting Engineers, Wellington, New Zealand.

Saliger[33] suggested that when torsion is resisted partly by reinforcement, partly by concrete, only the core enclosed by the hoops should be considered as effective concrete.

Recommendations related to combined torsion, transverse shear, and axial force would have been most useful. For relatively heavy axial compression the principal stress in the homogeneous section may give the limitations.[34] The combination of torsion and/or transverse shear and axial tension, however, usually presents difficulties.

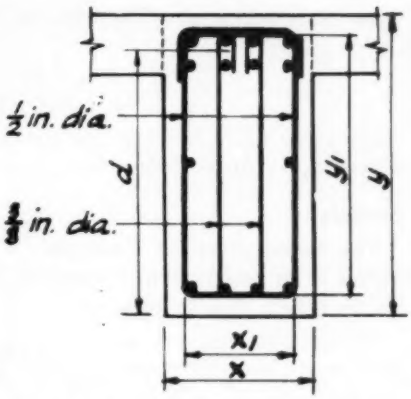


Fig. D—Combination of 1/2 in. and 3/8 in. diameter stirrups to resist torsion and transverse shear

When torsion and transverse shear act together and reinforcement is required for the whole or part of this load it is customary to calculate the necessary steel for each case and add the results. This is correct only if two legs are used for shear reinforcements as shown in Fig. 18. If transverse shear is large it will be necessary to provide more than two legs through the web of a beam in which case the simple superposition does not hold. It may be assumed that the steel stress in a leg is proportional to its distance from the center of the beam. Obviously inner legs are less effective in resisting torsion and as a result they can never be fully stressed under a combination of torsion and transverse shear, without the outer legs being overstressed. Convenient equations can be derived for various arrangements of shear reinforcement, two of which are shown here:

1. A combination of ½-in. and ⅜-in. diameter stirrups is used as indicated in Fig. D. The following assumptions and simplifying substitutions are made: (a) inner legs do not contribute to torsional resistance; (b) d = approx. y_1; (c) j = ratio of distance between centroid of compression and centroid of tension to the depth d = approx. 0.87; (d) $k = M_{ts}/x_1V$; (e) V = transverse shear force; and (f) n = number of stirrup legs used in one cross section. If the maximum steel stress in the most adversely affected stirrup leg is limited to f_s, the spacing of stirrups along the axis of the beam will be

$$s = \frac{f_s\, d}{V}\left(\frac{0.314}{k + \dfrac{3.27}{n + 1.56}}\right)$$

2. The diameter of all stirrup legs is the same, but for the outer hoop a double bar is used (i.e., outer legs are twice as close to each

other as inner legs). The spacing of this set of stirrups will be

$$s = \frac{f_s a_s d}{V}\left(\frac{3.20}{k + \frac{3.68}{n+2}}\right)$$

where a_s = area of one bar.

Example

The beam given in Example "6" is subject to a transverse shear of 25,000 lb in addition to a twisting moment of 100,000 in.-lb.
Shear stress due to torsion

$$v_t = \frac{100{,}000}{0.246 \times 12^2 \times 24} = 118 \text{ psi}$$

Shear stress due to transverse shear force

$$v = \frac{25{,}000}{12 \times 0.87 \times 21.5} = 112 \text{ psi}$$

$$v_{max} = 118 + 112 = 230 \text{ psi}$$

It is advisable in this case to provide reinforcement for the whole of the torsion and transverse shear. If the main tensile reinforcement consists of four bars, four legs could be used for the stirrups. If the outer legs are ½ in. diameter, the inner legs ⅜ in. diameter bars, the spacing of this set when $n = 4$ is obtained thusly:

$$k = \frac{100{,}000}{9.5 \times 25{,}000} = 0.42$$

$$s = \frac{18{,}000 \times 21.5}{25{,}000}\left(\frac{0.314}{0.42 + \frac{3.27}{4 + 1.56}}\right) = 4.83 \text{ in.}$$

If three legs are used for the stirrups in the following sizes ½ in., ⅜ in., and ½ in., the spacing would be

$$s = 4.26 \text{ in.}$$

Finally, if just a single ½ in. diameter hoop is used, the spacing, by putting $n = 2$, would be

$$s = 3.63 \text{ in.}$$

The percentage of vertical steel per unit length of beam would be, for the above three cases, respectively

$$(2 \times 0.196 + 2 \times 0.110) \times \frac{12}{4.83} \times \frac{100}{12 \times 24} = 0.53 \text{ percent}$$

$$(2 \times 0.196 + 1 \times 0.110) \times \frac{12}{4.26} \times \frac{100}{12 \times 24} = 0.49 \text{ percent}$$

$$(2 \times 0.196) \times \frac{12}{3.63} \times \frac{100}{12 \times 24} = 0.45 \text{ percent}$$

This indicates the inefficiency of inner stirrup legs, the use of which, however, can not be avoided in many cases.

REFERENCES

31. E. Rausch, *Drillung (Torsion) Schub und Scheren im Stahlbetonbau*, Deutscher Ingenieur-Verlag GmbH, Dusseldorf, 1953.

32. "Bauwerke aus Stahlbeton—DIN 1045," The German Concrete Code.

33. R. Saliger, *Der Stahlbetonbau*, 7th Edition, Franz Deuticke, Vienna, 1949.

34. "Regles d'Utilisation du Béton Armé Applicables aux Travaux Dépendant du Ministère de la Reconstruction et de l'Urbanisme et aux Travaux Privés," The French Concrete Code, 1948.

By PAUL ZIA*

Professor Cowan and his associates have indeed made a significant contribution to the engineering profession through their research on torsion in concrete structures. This is evident from the number of references on the subject in his present paper. In this paper Professor Cowan has drawn results from his previous researches and clearly presented design formulas for reinforced concrete beams subject to torsion. This discussion is primarily concerned with certain details regarding the torsional shear reinforcement.

The author has rightly stated that the most practical torsional shear reinforcement in a rectangular section is a combination of closed hoops and longitudinal reinforcement. The hoops and the longitudinal bars are to resist the vertical and the horizontal components, respectively, of the diagonal tension in concrete caused by torsion. Since the maximum torsional stress in a rectangular section occurs at the middle of the longer sides and the stress is negligibly small at the corners, the logical location for the longitudinal reinforcement is at the middle of each side. The Australian code, on the contrary, specifies one bar to be placed at each corner of the hoop as the minimum requirement. The weakness in placing the longitudinal bars at the corners rather than at the middle of each side of the rectangle lies in the fact that the hoops may not develop their potential strength because of the lack of resistance to the horizontal component of the diagonal tension which causes first cracking at the middle of the longer sides. This fact was observed by the writer in his recent investigation of torsional strength of prestressed concrete members. (The results of this investigation will be presented in a forthcoming paper.)

*Member American Concrete Institute, Assistant Professor of Civil Engineering, University of Florida, Gainsville, Fla.

The other important detail of the torsional shear reinforcement is the maximum spacing of the hoops which the author did not mention as a requirement in the new Australian code. It is obvious that if the spacing of the hoops is equal to or greater than the depth of a rectangular beam, a diagonal crack due to torsion may form between the hoops. It appears desirable, therefore, that the spacing of the hoops be limited to not more than perhaps 0.40 of the beam depth. Similar requirements are in the ACI Code regarding the design of stirrups for bending shear.

Discussion of a paper by Geoffrey Brock:

Effect of Shear on Ultimate Strength of Rectangular Beams with Tensile Reinforcement*

By C. BERWANGER and E. S. MAGILL, R. B. L. SMITH, and AUTHOR

By C. BERWANGER and E. S. MAGILL†

We agree with Mr. Brock's comments about the normal limitations regarding research dealing with reinforced concrete. The results of research in concrete and reinforced concrete lends itself especially to analysis by statistical methods. It may even be said that reinforced concrete is essentially a statistical material and that its properties are largely dependent on the random variation of its components. The main variation being due to the concrete and smaller variations in the steel properties. It requires a large number of tests to produce sufficient data for adequate checking of any particular behavior or theory. It is felt that the statistical analysis of experimental data has not always been possible nor given sufficient importance in research work dealing with reinforced concrete. With the good control possible in the research laboratory, and the greater reproducibility of results, it is possible to reduce the number of tests required somewhat. However, any full scale testing is expensive. It would seem that the use of plaster models would present a possible method by which the normal test difficulties and cost of testing full-size reinforced concrete beams might be overcome.

However, it is felt that perhaps a more extensive program of testing would be necessary to fully determine the properties of the reinforced plaster model beams, which might be costly. There is also a difference in material to be considered. The plaster being homogeneous and the concrete heterogeneous. The behavior of the plaster beams may not be entirely similar to that of the reinforced concrete beams requiring a correlation of properties.

The reinforced plaster model beams would be of great value in any preliminary testing. The results of such model tests could be used in the planning of a limited series of tests with reinforced concrete prototypes.

*ACI JOURNAL, V. 31, No. 7, Jan. 1960 (*Proceedings* V. 56), p. 619. Disc. 56-37 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 32, No. 3, Part 2, Sept. 1960 (*Proceedings* V. 56).

†Members American Concrete Institute, respectively, Assistant Professor and Associate Professor, Department of Civil Engineering, University of Manitoba, Winnipeg, Man., Canada.

TABLE A—COMPARISON OF ULTIMATE MOMENTS

Year	Beam No.	f_c', psi	Experimental		Whitney Eq.	Fig. 12—a/d=1.67	p/p_o
			M_u, ft-kip	$M_u/f_c'bd^2$	$M_u/f_c'bd^2$	$M_u/f_c'bd^2$	
1951-52	1	2540	49.6	0.205	0.272	0.168	0.835
	2		80.9	0.332	0.272	0.168	0.835
1952-53	1	2860	100.4	0.366	0.246	0.160	0.735
	2		55.8	0.203	0.246	0.160	0.735
1953-54	1	2220	33.9	0.160	0.299	0.178	0.950
	2		43.8	0.206	0.299	0.178	0.950
1954-55	1	2400	57.4	0.250	0.281	0.172	0.875
	2		47.0	0.204	0.281	0.172	0.875
1956-57	1	2860	65.6	0.240	0.246	0.160	0.735
	2		75.2	0.275	0.246	0.160	0.735
1957-58	1	2600	84.2	0.338	0.265	0.166	0.808
	2		55.9	0.224	0.265	0.166	0.808
1958-59	1	2500	77.0	0.322	0.272	0.168	0.838
	2		55.0	0.230	0.272	0.168	0.838
1959-60	1	3370	69.3	0.214	0.216	0.150	0.625
	2		67.5	0.209	0.216	0.150	0.625

It is also felt that the application of the ultimate load design method to reinforced concrete beams and structures is not entirely dependent on the shear consideration provided that adequate provision is made for web reinforcement to resist the full shear at ultimate load. The main consideration in the ultimate load or limit design method is a ductile material, or more precisely a material that is sufficiently ductile for the particular loading and span.[10-11] It is essential that failures other than the ductile collapse required be designed against. Thus critical sections of a structure should not fail in compression, or through shear, bond, or anchorage deficiencies.

In Table A the results of a series of simply supported reinforced concrete beams, loaded at the third points, is presented and compared to data from the design chart in Fig. 12 of the paper and to Whitney's equation. The experimental data presented has been derived from the reinforced concrete beams tested by the final year civil engineering students at the University of Manitoba, during the past several years, under the direction of Prof. E. S. Magill. The information is not as complete as could be desired, the crack patterns not being available in most cases, but there seems to be reasonably good correlation.

The beam cross sections were 8 x 14 in. with an effective depth of 12 in., 6 ft long supported on a 5 ft span. Tension reinforcement, in all cases, consisted of 2–⅝ in. diameter and 2–¾ in. diameter deformed steel bars having an average yield strength of 55,000 psi and without web reinforcement. Beam 1 had straight bars for the full length of the beam and Beam 2 had the same bars hooked.

REFERENCES

10. Berwanger, C., Application of Plastic Design Theory to Reinforced Concrete Continuous Beams," MS Thesis, Queen's University, Kingston, Ont., 1957.

11. Berwanger, C., and Wright, D. T., "Plastic Redistribution of Moments in Reinforced Concrete Beams." Paper presented at the ASCE Los Angeles convention, Feb. 1959.

By R. B. L. SMITH*

The new method of experimental research in reinforced concrete outlined by Dr. Brock is of great interest and significance, and in this paper, as in his previous one on the use of plaster models, his main task has been to establish the validity of the method, since much of his actual treatment of shear effects in reinforced concrete beams has already been discussed elsewhere.[3,12]

As stated by Dr. Brock, the importance of this new approach arises from the difficulties associated with the systematic investigation of any particular variable in reinforced concrete. In using data assembled from the results of tests carried out by others, there are uncertainties in the relation of cylinder or cube strengths to the concrete strength in the beams, due to variations in compaction and curing conditions, shrinkage effects, and methods of load application. Such uncertainties are inherent in practice: design methods must recognize them in the evaluation of load factors and it is essential to analyze all available data with this object, but for systematic research the closest possible control is required.

Dr. Brock offers a solution to this problem. The degree of control depends on the basic consistency of the model materials, high quality plaster and steel, and on the manufacture and strength correlation for various mixes. The extremely small scale of the models would be expected to cause difficulty in the simulation of details such as web reinforcement, but the technique should prove extremely valuable for the investigation of building frames and also for tests on the buckling of beams.

The present writer has, in collaboration with F. A. Hammond, recently described[15] an alternative solution: the elimination of the concrete strength variable within a set of tests by casting all the specimens in one operation, curing and testing under identical conditions, except for the variable under investigation; also the use of a subsidiary test on each specimen as a check on consistency.

While agreeing with the Author's definition of a flexural failure as one in which the full moment of resistance is developed, even if the appearance of the beam suggests shear failure, it is felt that the type of failure illustrated as flexure in Fig. 5 is only valid for small ratios of p/p_o in which the diagonal crack does not reduce the compression zone below the area necessary for flexural resistance. Larger p/p_o ratios would bring about "shear-compression" failure (as defined by Laupa)[3] but due to the lack of web reinforcement, splitting caused by dowel action[13] may produce the premature failure described by Dr. Brock as

*Lecturer, Structural Engineering Department, Manchester College of Science and Technology, Manchester, England.

shear bond. This splitting type of failure was discussed by the present writer, in a joint paper with K. Subbiah[14] and also in the discussion to Reference 3. It is difficult to predict with accuracy, since it depends on the resistance to splitting of the concrete at the level of the reinforcement between the bottom of a diagonal crack and the support. This accounts for the scatter of points in the triangle formed by the lines marked "flexural failure," "diagonal tension failure," and "shear bond failure" in Fig. 3 and 4. Due to the erratic nature of such failures, which do not occur in beams with stirrups, it is considered by the present writer that tests should be carried out on beams with stirrups, even though this means the introduction of a further (albeit essential) variable into the test program. A beam without such web reinforcement is not properly a reinforced concrete beam.

REFERENCES

12. Whitney, Charles S., "Ultimate Shear Strength of Reinforced Concrete Flat Slabs, Footings, Beams, and Frame Members Without Shear Reinforcement," ACI JOURNAL, V. 29, No. 4, Oct. 1957 (*Proceedings* V. 54), pp. 265-298.

13. Watstein, D., and Mathey, R. G., "Strains in Beams Having Diagonal Cracks," ACI JOURNAL, V. 30, No. 6, Dec. 1958 (*Proceedings* V. 55), pp. 717-728 and discussion in V. 30, No. 12, June 1959 (*Proceedings* V. 55), pp. 1451-1458.

14. Subbiah, K., and Smith, R. B. L., "Influence of Shear on the Moment of Resistance of Reinforced Concrete Beams," *The Structural Engineer* (London), V. 36, No. 11, Nov. 1958.

15. Hammond, F. A., and Smith, R. B. L., "A Preliminary Study of Moment-Shear Interaction in Reinforced Concrete Beams," *Civil Engineering and Public Works Review* (London), V. 55, No. 647, June 1960.

AUTHOR'S CLOSURE

Most of the discussion has been concerned with the technique and use of reinforced plaster models. For the purpose of this paper they were merely an expedient for the rapid collection of first-hand information on the collapse of reinforced beams. This seemed necessary before attempting any reassessment of the tests on concrete beams which had been reported by previous investigators. The model tests can thus be regarded as complementary to the full-scale ones. This philosophy, though new to structural concrete research, has been current for some years in the study of steel structures and was succinctly described by Prof. J. F. Baker in his account of the development of plastic theory.[16] "It was clear from the beginning of the investigation, therefore, that owing to the expense involved, full scale structures could only be used to provide spot tests, their main purpose being to demonstrate to the engineer that the new method was reliable. The detailed knowledge of plastic behavior would have to be obtained from small-scale, but still real, structures..."

When we progress from steel structures to those of reinforced concrete, our investigations must include a further major variable, the

variety of ways in which the reinforcement can be arranged. Our lack of real knowledge of the relative effectiveness of different reinforcement details is a reflection of the complexity of this additional problem. The scope of the paper was limited to the simplest arrangement, rectangular-section beams having only straight fully-anchored reinforcement, which had also been the one used in most of the earlier full-scale tests. However, the writer would not suggest that this is the best way of using main reinforcement, and the discussers have pointed out that web reinforcement is usually employed in practice. But a treatment of these important practical modifications is rather outside the scope of the paper. Tests on beams without web reinforcement are useful because we cannot assess its effectiveness unless we know the performance of those without it. It is also worth remembering that rectangular beams are relatively uncommon in isolation, being usually encountered as elements of slabs where web reinforcement is not often provided.

The writer would agree with Professor Berwanger and Magill that a minimum degree of ductility is required for the application of ultimate load methods to continuous structures. However, depending on the geometry of loading, the span-depth ratio and the reinforcement index p/p_o, it is possible that it will be obtainable without web reinforcement. Some data for two-span beams under uniformly distributed load was obtained by Fowler, Hohenkerk, and Leake and is given in Fig. C of the discussion of a recent paper by Rodriguez, Bianchini, Viest, and Kesler.[17] It is unfortunate that the ASCE paper by Berwanger and Wright was not available at the time of writing this closure.

Mr. Smith's technique of mass-producing real concrete specimens forms a useful method of obtaining sufficient experimental results to define both the trend and the scatter with such a variable material. He is also to be thanked for drawing attention to the 1957 paper by Whitney, from whose well known earlier work on flexure the present paper has been developed. In comparing the two methods, it is apparent that Whitney's method will classify as shear failures many which the present writer would regard as flexural. For example, it is possible to compare the minimum values of L/d required to produce flexural failures in simple beams with "balanced reinforcement."

Midspan load:

Whitney	$L/d = 12.9 - 18.5$, depending upon f_c'
Author	$L/d = 4.5 - 10.0$, depending upon f_c'

Uniformly distributed load:

Whitney	$L/d = 15.0 - 20.5$, depending upon f_c'
Author	$L/d = 6.0 - 10.0$, depending upon f_c'

As a consequence of this difference, the tests under uniform load reported by Bernaert and Siess[9] are regarded as examples of flexural failures by the writer.

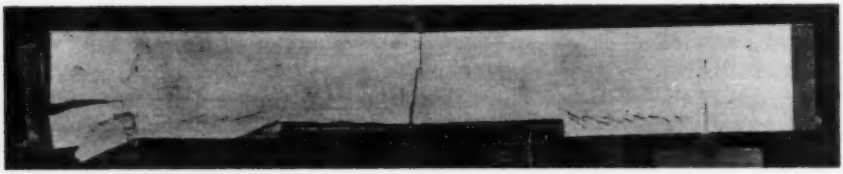


Fig. A—Shear-bond cracks in beam without dowel action

Dowel action is suggested as a possible cause of the horizontal cracks associated with the shear-bond type failure. Following up this suggestion by Mr. Smith, the writer made tests of the kind shown in Fig. A. Here the central portion of the reinforcement has been left exposed to prevent any dowel action, but the pattern of horizontal cracks just above the reinforcement is unchanged. Eccentric pull-out tests, designed to simulate beam conditions, also confirm that dowel action is not the cause of these cracks.

It appears to the writer that such cracks can best be attributed to the combined effects of horizontal shear and longitudinal tension. The horizontal shear concept was common in reinforced concrete design at the beginning of the century until it was "ruled out" by the advent of various national rules. But the theory does not appear to have been disproved any more than the Morsch theory has ever been properly justified, and it is interesting to find it still appearing in a textbook written as recently as 1953.[18]

The results presented by Professors Berwanger and Magill have been superimposed on Fig. 13 and reproduced as Fig. B. Allowing for the big scatter, natural in tests spread over such a long time period, they fit

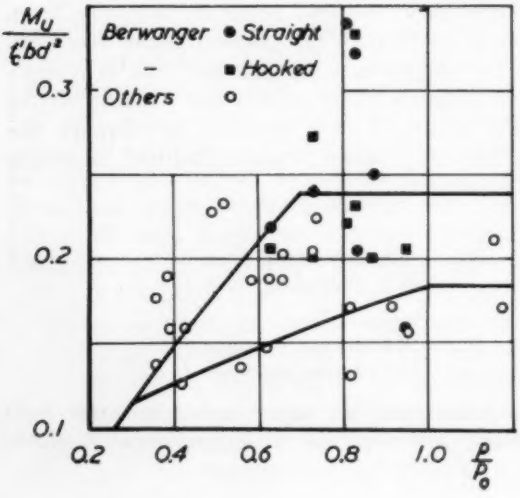


Fig. B—Results of tests on reinforced concrete beams ($a/d = 1.75$)

fairly well to the pattern of the other investigations. It is interesting to note that the beams with hooked bars were by no means always stronger than their companion specimens with straight bars.

REFERENCES

16. Baker, J. F.; Horne, M. R.; and Heyman, J., *The Steel Skeleton. V. II—Plastic Behaviour and Design*, Cambridge, 1956.

17. Brock, G., Discussion of "Shear Strength of Two-Span Continuous Reinforced Concrete Beams," ACI JOURNAL, V. 31, No. 6, Dec. 1959 (*Proceedings* V. 56), pp. 1571-1575.

18. Eriksen, B., *Theory and Practice of Structural Design Applied to Reinforced Concrete*, Concrete Publications, London, 1953.

Discussion of a paper by Elihu Geer:

Stresses in Deep Beams*

By ARTURO M. GUZMÁN and AUTHOR

By ARTURO M. GUZMÁN†

The analysis of deep beams is always a problem of considerable practical importance. The paper by Professor Geer is welcome as it gives us new information regarding different load situations.

The author uses the method of finite-differences, but with square gridwork, taking into account the considerations which the writer expressed in his discussion of the paper by Chow, Conway, and Winter.[14, 15] That way exactitude is improved considerably. In the same way the stresses are calculated using the five-point formula for second derivatives, by which method the tensions, especially in boundaries, receive a better determination.

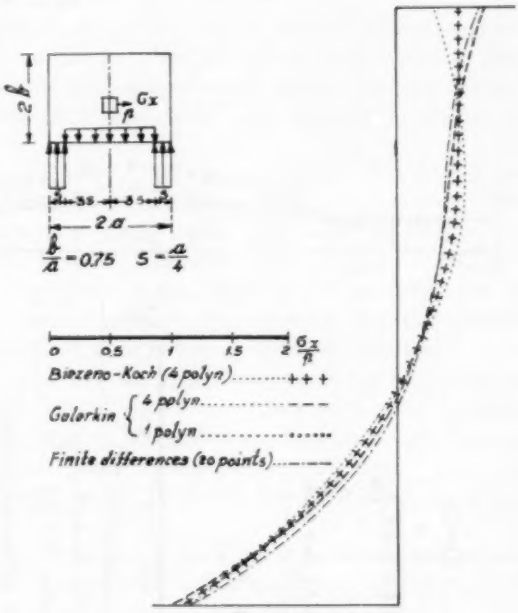


Fig. A

As I pointed out in discussion,[15] another alternative and more direct solution (four equations with as many unknowns) may be obtained by a variational way. The writer has developed in a recent paper the ap-

*ACI JOURNAL, V. 31, No. 7, Jan. 1960 (*Proceedings* V. 56), p. 651. Disc. 56-39 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 32, No. 3, Part 2, Sept. 1960 (*Proceedings* V. 56).

†Professor of Civil Engineering, Facultad de Ciencias Exactas y Tecnologia, Universidad Nacional de Tucumán, San Miguel de Tucuman, Argentina.

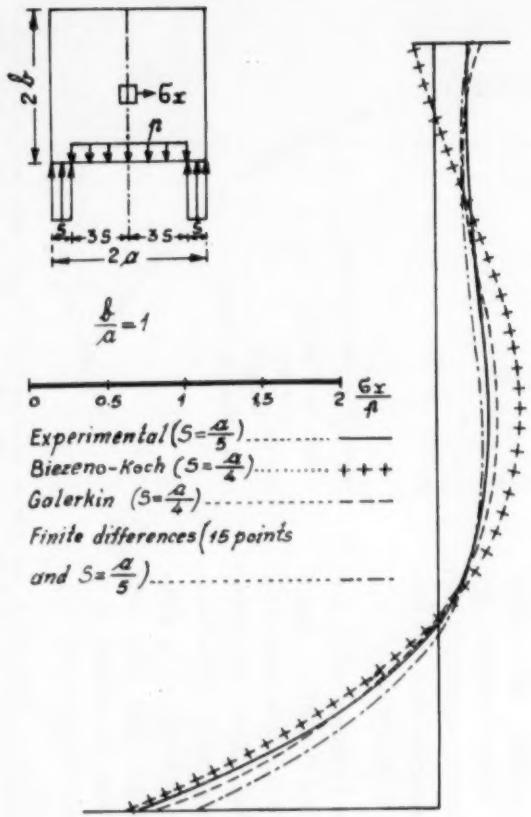

2b
σx
p
s
3s
3s
s
2a
b/a = 1
0
0.5
1
1.5
2
σx/p
Experimental (S = a/5)
Biezeno-Koch (S = a/4)
Galerkin (S = a/4)
Finite differences (15 points and S = a/5)

Fig. B

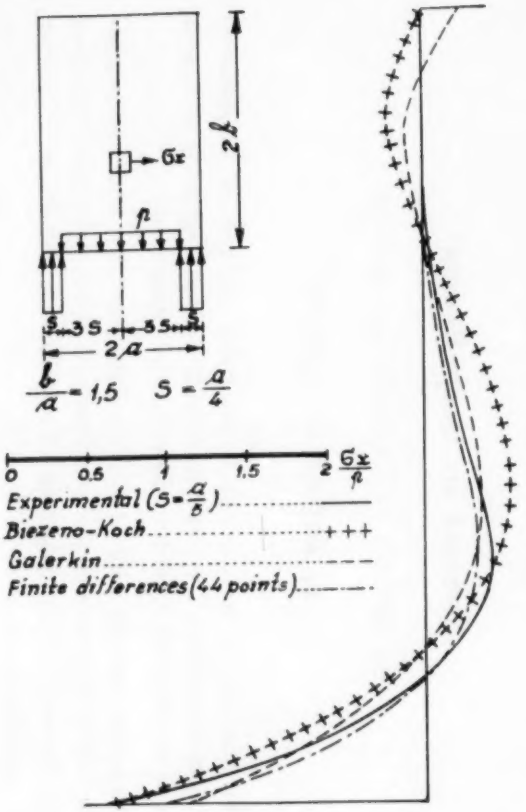

2b
σx
p
s
3s
3s
s
2a
b/a = 1,5
S = a/4
0
0.5
1
1.5
2
σx/p
Experimental (S = a/5)
Biezeno-Koch
Galerkin
Finite differences (44 points)

Fig. C

proximation which can be obtained by the method of Galerkin and Biezeno-Koch,[24] comparing them with the finite-differences for the case of a single span deep beam with the load at the bottom. Fig. A, B, and C show the curves for the horizontal stress σ_x along the center line of span. The experimental values in Fig. B and C were obtained by photoelasticity. The differences are of little importance in Galerkin and finite-differences methods, but Biezeno-Koch method gives higher values, and is evidently a method that leads to an upper boundry. In Fig. A Galerkin's method was employed with one of four polynomials to investigate the convergence.

The author's statement that "the greatest tensile stress occurs not at the midspan but near the face of support" was given by the writer employing Galerkin's variational method, as hitherto the method of finite-differences did not reveal this fact because of the coarseness of the net employed. The author explains this well by Fig. 7.

All the above discussion refers to the pure mathematical analysis in the elastic state. It is also interesting to study the experimental behavior in the rupture, as hitherto little research work has been performed. Photoelastic investigations are more numerous but these refer also to the elastic aspects.

To increase the bibliography given by the author, the most fundamental of which is the paper by Nylander and Holst,[18] the writer wants to add some publications which permit the estimation of safety referent to the ultimate load, as well as the most favorable arrangement of the steel reinforcement.

The work done by Klingroth,[25] by Graf, Brenner, and Bay[26] and recently by Schütt[27] may be mentioned, as their performances indicate the trend for future experimentation, using little specimens of mortar or concrete with different types of reinforcement.

It is necessary to stress again the importance of the compression at the supports, as there may occur a rupture by crushing of the concrete. In every case there exists at this point a heavy local stress, and cracks always start in this region. As accentuated already by Schütt the cracks along the span do not become worse, meanwhile the cracks near the support open more and more. The author verifies a rupture by shear. On the other hand, the paper of Schütt confirms the above statement[15] that the stress of the reinforcement at rupture is much less than that accorded to it by the elastic theory.

More research work on this problem is necessary to determine safely the collapse load of deep beams, as well as an economic calculation of the reinforcement.

REFERENCES

24. Guzmán, A. M., and Luisoni, C. J., "Sobre la Aproximación de Algunos Métodos Variacionales en Elasticidad" (On the Approximation of some Varia-

tional Methods in Elasticity) (Spanish, with an English summary). *Ciencia y Tecnica* (Buenos Aires), V. 125, No. 628, 1958.

25. Klingroth, "Versuche an Stahlbetontragwänden," *Beton und Stahlbetonbau* (Berlin), No. 9-14, 1942.

26. Graf, Brenner, and Bay, "Versuche mit einem wandartigen Träger aus Stahlbeton," *Bulletin* No. 99, Deutscher Ausschuss für Stahlbeton, 1943.

27. Schütt, H., "Über das Tragvermögen wandartiger Stahlbetonträger," *Beton und Stahlbetonbau* (Berlin), No. 10, 1956.

AUTHOR'S CLOSURE

The author is appreciative of the discussion from Professor Guzmán which supplements the original paper and also appreciates the various comments he received from JOURNAL readers through correspondence.

Discussion of a paper by Edward K. Rice and Felix Kulka:

Design of Prestressed Lift Slabs for Deflection Control*

By ARTHUR M. JAMES, M. SCHUPACK, and AUTHORS

By ARTHUR M. JAMES†

The authors do an excellent job of presenting the design of a simple, regular flat plate prestressed lift slab. Perhaps their most significant point is the remark on p. 687 under the heading *Loading* ". . . the loading used for design must be chosen logically." What this means, of course, is that it is foolish to prestress for a seldom experienced live load, and create a camber problem, perhaps a greater monster than the deflection problem which the prestress is trying to prevent.

A classic example of prestressing for a seldom applied live load is, of course, our present bridge design code. Being forced to prestress, for zero tension under the occasional combinations of full design live loads, plus impact, we of necessity build huge upward camber creating forces into our beams.

I well remember when we redesigned the Kenai Bridge in Alaska into lightweight concrete units fabricated in Portland, Ore.‡ Because of the requirements imposed by the Bureau of Public Roads (for 39 percent prestressing losses) there was an 1⅝ in. initial camber built into the 70-ft girders. Several months later these girders arrived in Alaska with an average of almost 3¼ in. of camber. The precast lightweight deck slabs took a little of this out but it was still a thrill for the natives to hit the four 70-ft spans and enjoy the roller coaster effect. I should also add that the blacktop wearing slab was also omitted; of course the dead load would have been welcomed as well as the levelling effect.

This example illustrates, I think, the lack of wisdom in designing for zero tension under unusual loads and on the other hand the wisdom and appropriativeness of designing for the more ideal state of zero or close to zero camber (uniform P/A on the cross section under real permanent loads, and for a reasonable factor of safety under ultimate loads).

*ACI JOURNAL, V. 31, No. 8, Feb. 1960 (*Proceedings* V. 56), p. 681. Disc. 56-40 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 32, No. 3, Part 2, Sept. 1960 (*Proceedings* V. 56).

†Member American Concrete Institute, Consulting Engineer, Portland, Ore.

‡ACI JOURNAL, V. 26, No. 10, June 1955 (*Proceedings* V. 51), pp. 1025-1034.

In the authors' article, they do not state whether the tensioning units are bonded or unbonded; if unbonded, as I suspect, then there should be mild steel to take the tension referred to under live load, or at least further analysis to justify the use of Whitney's and the ACI adopted theory of ultimate strength design. The entire proof of ultimate strength capacity in Whitney's excellent theory is based on bonded steel continuing to elongate under continuing increases of load and further strains. I find no justification for the convenient assumptions of p. 690, under Ultimate Strength, which seems to assume that unbonded prestressed concrete follows the same route to the same ultimate strength as bonded prestressed concrete or reinforced concrete. I think this is an extremely dangerous conclusion which the authors probably *did not intend* to make. It should be clarified before some others who have not thought the matter through, accept the conclusion, and get in trouble with it.

SUMMARY

Design for minimum camber, or partial prestressing, particularly where excessive camber is objectionable. In structures subject to heavy, occasional live loads, this is particularly true.

Adequate ultimate strength must be provided.

The implied assumption that the unbonded prestressing follows Whitney's theory to the same ultimate strength is unproved and therefore may be dangerous.

By M. SCHUPACK*

This paper lucidly points out the need for designing structures for performance and not blindly, such as designing only for stresses as required by the code. The need for this philosophy of design is becoming increasingly important, particularly as the concepts of ultimate design, plastic design, and/or limit design become more accepted.

Referring to p. 690 it would be well for the authors to explain how a greased (unbonded) tendon can be assumed to fail at 210,000 psi. It is not unlikely that in many cases the stress in the tendon may be appreciably below the 210,000 psi range when failure will occur. At best, predicting ultimate strength of a slab with an unbonded tendon with the limited test data available, is an approximation. It would be helpful for designers to have additional data available in this regard.

The difficulty in accurately determining the steel stress was discussed in the ACI-ASCE Committee 323 report, "Tentative Recommendations for Prestressed Concrete."[6]

*Member American Concrete Institute, Consulting Engineer, Schupack and Zollman, Stamford, Conn.

It may be well to repeat here the paragraph that precedes their recommendations for maximum allowable steel stress at ultimate capacity:

> "(b) *Unbonded members* — Ultimate flexural strength in unbonded members generally occurs at lower values of steel stress than in bonded members. Wide variations between stress levels reported by different investigators reflect the fact that several factors influence the stress developed by unbonded steel at ultimate moment. These factors include: magnitude of effective prestress, profile of the prestressing steel, shape of the bending moment diagram, length-depth ratio of the member, magnitude of the friction coefficient between the prestressing steel and duct, and amount of bonded non-prestressed supplementary steel."

Another matter which needs more study is the minimum normal reinforcing steel which should be used for this type of construction. It is my understanding that some lift slabs have had none or next to no normal reinforcement. Having minimum percentage of reinforcing may minimize shrinkage cracking and certainly will increase ultimate strength or minimize localizing cracking.

AUTHORS' CLOSURE

The authors wish to thank Messrs. James and Schupack for their thoughtful comments. Particularly gratifying are the writers' agreement on the logical approach to prestressed concrete design as opposed to the blind following of codes of practice.

Both Messrs. Schupack and James seemed to be concerned about the application of Whitney's theory to ultimate design of prestressed slabs. The Whitney theory assumes rectangular distribution of stresses in concrete at the ultimate range. While it was developed for bonded reinforcement it has been proven that it is close enough for prestressed sections, whether bonded or not. The only difference lies in the calculation of the stress in the tendon at the ultimate moment; where, of course, the stress in the unbonded tendon is lower and where one must not use the ultimate stress.

The existing stress in the unbonded tendons at the ultimate moment is treated in several papers. The authors wish to point out in particular experiments by Scordelis, Pister, and Lin[7] as well as Lin, Scordelis, and Itaya[2] in which the tendon stress at ultimate moment was between 200,000 and 210,000 psi; the value of 210,000 psi is used in the authors' paper. It is true, however, that the tendon stress at failure in beams is lower than in slabs and in that case the specifications listed in the recommendations by Committee 323[6] may apply.

Mild steel reinforcement in prestressed slabs is added only at end anchorages, around openings, at places of stress concentration, and where potential cracking may occur. Mild steel reinforcement in a prestressed member will act only after cracking, hence it should be added only where actually needed, rather than indiscriminately.

Discussion of a paper by Roger Diaz de Cossio and Chester P. Siess:

Behavior and Strength in Shear of Beams and Frames Without Web Reinforcement*

By GEOFFREY BROCK, A. COÜARD, C. W. THURSTON and W. J. KREFELD, and AUTHORS

By GEOFFREY BROCK†

The data presented in this paper has been of considerable interest to the writer on account of its application to his method of predicting the strength of reinforced beams.‡ Generally the appearance of the failures shown in the photographs and line drawings of the paper corresponds with that of similar reinforced plaster models. However, the identification of modes of failure gives rise to some difficulty. For example, Fig. 7 shows apparently identical crack patterns for two beams, yet one is termed diagonal tension and the other shear-compression. The writer would prefer to describe both as shear-bond failures, the distinguishing feature being the horizontal crack at the level of the reinforcement. The term shear-compression seems to imply some deficiency which might be cured by employing compressive reinforcement because of the analogy with the term flexural-compression. However, experiments show that the addition of such reinforcement does not increase the strength of beams which fail in this way, thus indicating that there is no lack of compressive resistance.

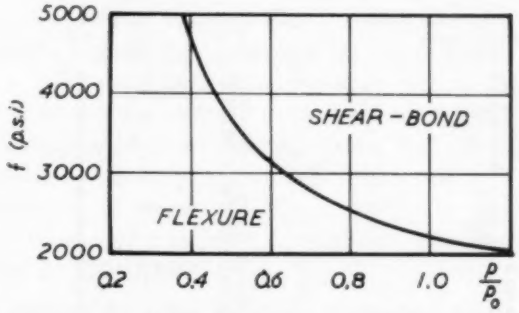


Fig. A — Chart predicting mode of failure

*ACI JOURNAL, V. 31, No. 8, Feb. 1960 (*Proceedings* V. 56), p. 695. Disc. 56-41 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 32, No. 3, Part 2, Sept. 1960 (*Proceedings* V. 56).

†Member American Concrete Institute, Department of Civil Engineering, University of Birmingham, Birmingham, England.

‡ACI JOURNAL, V. 7, No. 31, Jan. 1960 (*Proceedings* V. 56), pp. 619-638.

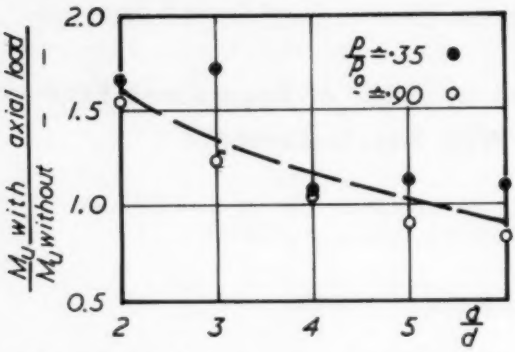


Fig. B—Effect of axial load on ultimate moment of beams

The numerical analyses made by the authors have concerned cracking load capacity, thereby seeming to depend considerably on the judgment of the observer. For this reason, and to fall in with the general concept of limit design, the writer prefers to investigate ultimate load capacity and has done so for the three groups of beam tests. The ultimate moments found in these tests have been predicted from Fig. 12 of Geoffrey Brock's paper,* using Fig. A (based on recent model beam tests) to separate shear-bond from flexural failures.

The results for beams and stub-beams under two point loads and without axial loads are given in Table A. The mean value of the ratio of actual to predicted ultimate moment is 1.10 with a coefficient of variation of 28 percent, if all the results are included. On discarding L-1 and its retest, which seem way out of line, these values become 1.02 and 15 percent respectively.

*Ibid.

TABLE A—BEAMS AND STUB-BEAMS UNDER TWO-POINT LOADS (WITHOUT AXIAL LOAD)

Beam mark	p/p_o	a/d	Predicted $M_u/f_c'bd^2$	$\frac{M_{obs}}{M_{pred}}$
L-1	1.06	2	0.19	1.52
L-2	1.06	3	0.23	1.20
L-2a	0.57	3	0.16	1.04
L-3	0.82	4	0.24	0.83
L-4	0.87	5	0.28	0.93
L-5	0.87	6	0.28	0.86
L-6	0.76	7	0.26	1.09
L-1R	1.06	2	0.19	2.16
L-2R	1.06	3	0.23	1.18
L-2aR	0.57	3	0.16	1.22
L-3R	0.82	4	0.24	0.97
A-1	0.36	2	0.13	1.13
A-2	0.33	3	0.12	0.86
A-3	0.51	4	0.18	1.07
A-4	0.38	5	0.13	1.21
A-5	0.36	6	0.13	1.19
A-11	0.86	2	0.18	1.05
A-12	1.03	3	0.23	0.75
A-13	0.91	4	0.26	0.85
A-14	0.90	5	0.29	0.88
A-15	0.84	6	0.27	1.11

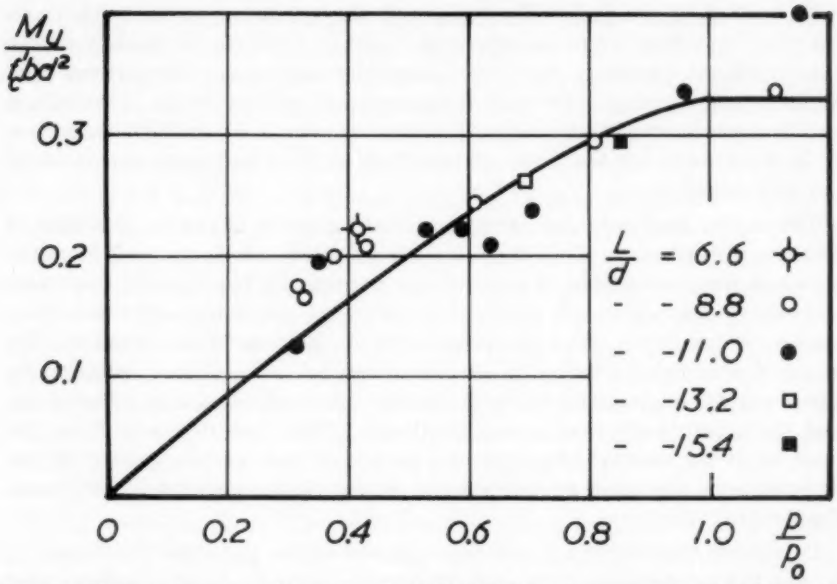


Fig. C—Tests on beams under uniformly distributed load compared with Whitney's flexural theory

The effect of the 20 kip axial load is not very clear from the limited number of tests. Pairing tests with and without axial load, and ignoring the fairly small differences of p/p_o within these pairs, we can produce Fig. B showing the trend of the effect of a/d.

The beams under uniformly-distributed load have been taken to be those previously reported in Reference 7 despite a discrepancy in the value of p for the last group of beams in Table 1. Fig. C shows the ultimate moment at midspan plotted against the value of p/p_o. These points are in good agreement with Whitney's curve for flexural resistance, thus indicating that shear had no adverse effect on the load capacity of this group. The same conclusion could be obtained from Fig. 10 of Brock's paper,* which indicates that smaller span-depth ratios would be necessary to demonstrate the effect of shear.

By A. COÜARD†

We must congratulate the authors for what we think is the best paper yet published on shear, without disparaging those already published and referred to which broke the ground.

*Ibid.
†Member American Concrete Institute, Consulting Engineer, Paris, France.

Nevertheless, we feel obliged to draw the attention of the authors to the fact that their experiments and findings seem to us mostly to be confirmations, priceless from our point of view, of our theoretical predictions and assumptions, since 1953 onwards, presented in *Génie Civil* and in the second edition (revised 1957) of our book *Nouvelle Conception de la Résistance des Matériaux* whose first edition had been reviewed in the ACI JOURNAL.

The main, and only criticism we would dare to make to the papers already published is that they try to establish an empirical formula which is not susceptible of general use because its basic parameters are the wrong ones: they are directly inspired by a nostalgia of the elastic theory, which is not valid philosophically in the case of solids where the angle of internal friction ϕ is $\neq 0$. The term bd has no direct relation to shear nor M; what matters are the compression stresses along the beam and the traction stresses across the beam. These are dependent on the position of the neutral fiber, which depends on the relative strains of the concrete and the main reinforcement, as the shear cracking "lock" is on the neutral fiber.

It is there that a correct and basic theory of the physical phenomenon is absolutely primary, because from the theory chosen depend the choice of the parameters to be considered, and if you choose the wrong ones, no formula can ever be devised which could hope to be valid in all cases, under all cases of loadings, and with every form of structure, and any material.

It seems that most present research work efforts could bear more fruit if more international cooperation and use of the work done all over the world on the same subjects was systematically pursued.

By C. W. THURSTON* and W. J. KREFELD†

The authors of this paper have made a valuable contribution to the problem of predicting the behavior of reinforced concrete beams subjected to shearing forces, and the presentation of their findings is welcomed by all researchers considering the same general topic. The writers of this discussion have been engaged in an investigation of simply supported beams without web reinforcement subjected to similar loadings, and would like to comment on some of the points raised in the paper which they also have encountered in their own observations and analyses.

In most of the published papers of the past decade discussing the "shearing" strength of reinforced concrete beams, the researchers have sought to identify and designate a particular condition of the cracking of the concrete which develops during the destruction of such beams

*Assistant Professor of Civil Engineering, Columbia University, New York, N.Y.
†Member American Concrete Institute, Professor of Civil Engineering, Columbia University, New York, N.Y.

when they are loaded to failure. Typically, this paper describes inclined tension cracks which result from combinations of increasing bending and shearing stresses, but reserves particular identification for these cracks when they become "fully developed." The load producing this particular condition is termed the "cracking load," and a related so-called "critical section" is also designated.

It is argued that the significance attached to the identification of such a cracked condition is justified because the internal mechanism by which the beam resists shearing forces changes markedly after this condition develops. Indeed, the attainment of this condition leaves many beams incapable of resisting any additional shear load; i.e., the cracking load is the ultimate load. The behavior of other beams also supports this view point, for the manner in which the cracks advance indicates that there is a definite development within the beam which occurs at a distinct point in its loading history. Yet for many beams this critical point is not readily recognized at the time it occurs during tests, but only in retrospect after observing the progress of the crack as loading continues.

Thus, as the authors point out, the "cracking load" is often a vague quantity, and different observers can report appreciably different values. Their differences may arise not only from difficulty in observing a change in the condition of the beam, but also from their having different concepts of what is causing this change. Published papers define various "cracking loads" in terms of those different aspects of beam behavior which the authors feel they should emphasize. This paper alone offers two separate definitions of "fully developed" inclined tension cracks, one on p. 696 and the other on p. 706. While both these definitions may be predicated on the observation of phenomena whose simultaneous occurrence is compatible, they are, nevertheless, by themselves independent. The question remains, what is the basic cause for the change in the condition of the beam?

The key to the degree of crack propagation may be contained in observations the authors made with electric strain gages placed on the concrete after the cracks had become "fully developed." They noted that in all beams on which such strain gages were placed it was found that the concrete below the inclined crack carried "an appreciable amount of compression." These results are confirmed by similar tests performed at Columbia where the gages were placed before the start of testing in 1, 2, or 3 vertical rows spaced 6 in. or more apart in regions where it was expected the diagonal cracks would eventually propagate.

How is the moment of the couple composed of the compressive force in the concrete below the diagonal tension crack and part of the tensile force in the longitudinal steel equilibrated? An analysis must include the moment of another couple, one of whose components is the vertical

force, commonly called dowel action, which the longitudinal steel exerts on the concrete near the point of intersection of the inclined crack and the steel. This vertical force eventually causes the horizontal (tensile) cracking of the concrete backward along the longitudinal steel toward the support. The start of this backward cracking corresponds with the condition described by saying the crack is "fully developed," and the degree of this cracking controls the propagation of the opposite end of the crack in the compression zone. Since compressive forces exist below the inclined crack until ultimate load is reached, a vertical dowel force must also be present.

Strain measurements taken during tests at Columbia on the top and bottom of the bars indicate local bending where the diagonal crack intersects the reinforcement, confirming the presence of dowel action forces. When the horizontal cracks form at the level of the bars the direction of the upper end of the diagonal crack changes, in most cases abruptly. The reported shape of typical diagonal cracks usually shows a "knee" after the crack has propagated to a point slightly above the neutral axis. This change in direction is directly related to the displacement resulting from the dowel action. While the vertical dowel forces probably decrease as the secondary cracking at the level of the bars proceeds, the bars themselves continue to offer some shear resistance. Regardless of the magnitude of this force, however, the vertical displacement of the separated parts across the lower end of the crack causes the rapid propagation of the crack in the compression zone and is critical to the subsequent behavior of the beam. In the case of relatively long span beams (large L/d), ultimate failure occurs immediately or soon after this displacement. For relatively short spans the ultimate load may be twice the "cracking load."

The authors have established empirical constants in their recommended equation (Fig. 16) which differ from those suggested by Viest.[3] The computation of these constants depends on the selection of the location of the assumed "critical section" and also on whether the equation predicts the average of actual test loads or a conservative lower limit of the spread in test results. The particular values of these constants and their corresponding critical section are not important provided they are consistent and predict the "cracking loads" accurately.

The Columbia tests also confirm the authors' observation that for beams under uniform loads the inclined crack intersected the longitudinal steel at an average distance of about 0.11 of the span from the support. For both uniform and concentrated loads, however, the Columbia tests (made on beams with the same sectional dimensions) gave consistently lower values of "cracking loads" than those predicted by the authors' equation. It is possible that the discrepancy may be attributed to the fact that the empirical constants were derived from tests on beams many

of which had external stirrups either immediately interior to or exterior to the end supports, whereas the Columbia beams provided sufficient overhangs for anchorage without end stirrups. The stirrups may have raised the value of the load at which horizontal cracking due to dowel action was initiated.

AUTHORS' CLOSURE

The authors wish to express their gratitude to Messrs. Coüard, Brock, and Thurston and Krefeld for their discussions. Their ideas throw additional light on the much-discussed problem of shear strength.

The authors appreciate the comments of Mr. Coüard. It is unfortunate that Mr. Coüard did not apply his theory to some of the actual test results reported in the paper, since his theory would then have become better known to the English-speaking public.

With regard to Mr. Brock's comments, the use of the term shear-bond instead of shear-compression is only a matter of nomenclature and definition, since neither compression nor bond were primary causes of failure. Using the ultimate load capacity of the members instead of the cracking load has obvious advantages for those members in which the tensile steel is near the yield point at failure, since then the steel stress is known, and the calculation of the moment capacity is only a matter of statics. However, this has not been the case for most members failing in shear, and the problem of their capacity remains a difficult one to solve, mainly because we still lack a complete understanding of the phenomenon. Mr. Brock is concerned only with the load capacity of the beams. However, ductility is a most important consideration in the behavior of a beam, and it is from this point of view that the differences between flexural and shear failures are most striking. Even though in some cases the flexural tension load may be reached at the same time as the cracking load, and the member develops a large amount of deformations before final collapse (as in Beam D-4 of the paper), the more common shear failure, occurring before yielding of the tensile steel, is a rather brittle type of failure and the deflections are very small.

Regarding the comments of Professor Thurston and Krefeld, perhaps it is not emphasized enough in the paper that the cracking load was considered to be reached when a fully developed crack formed, thus causing a redistribution of steel strains. The two concepts were never intended to be independent. If the inclined crack reaches the tensile steel, then, by statics, a redistribution of steel strain must occur. The discussion of the effects of the dowel force is lucidly written; the existence of this force cannot be denied, but its importance in relation to the design of members without web reinforcement must remain a question deserving further study.

The authors cannot explain why the "cracking loads" obtained at Columbia were consistently smaller than those reported in the paper, since it is not known exactly what criteria were used in the Columbia tests for determination of these loads. The authors urge Professors Thurston and Krefeld to publish their test results in complete detail since they will undoubtedly add to our still insufficient knowledge of shear strength.

Discussion of a paper by M. A. Gouda:

Distribution of Torsion and Bending Moments in Connected Beams and Slabs*

By H. S. GEDIZLI, AMIN GHALI, H. KLASMER, THOMAS PAULAY, A. SIEV, M. YEHYA, and AUTHOR

By H. S. GEDIZLI†

This discussion will be restricted to the presentation of some complementary data and references. The subject of the paper is very important and has been analyzed by various authors. To the writer's knowledge, it also is a new and original attempt at solving this problem as was made by Saeger.[8]

In the paper the torsional center is supposed to be the same as the center of gravity S of the beam cross section (Fig. A). This supposition has been used before by others,[9-12] and is claimed to be a good approximation for practical purposes. Actually the torsional center is at $\overline{S}$ (Fig. A) and, as indicated by Dr. Saeger, the final end moments at the edge of plate will be balanced by two moments in beam, namely; (1) torsional moments, and (2) horizontal bending moments.

The magnitudes of these two induced moments vary with the beam end restraints, beam span, and geometric shape of its cross section, i.e., with the dimensions of d, b, and η (Fig. A).

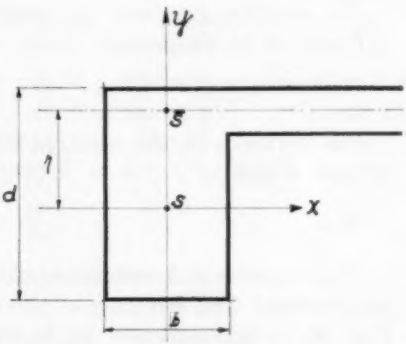


Fig. A—Torsional center of beam s

*ACI JOURNAL, V. 31, No. 8, Feb. 1960 (*Proceedings* V. 56), p. 757. Disc. 56-43 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 32, No. 3, Part 2, Sept. 1960 (*Proceedings* V. 56).

†Civil Engineer, Ankara, Turkey.

Furthermore, and especially in two-way reinforced concrete slabs, the shear in the plate seriously affects the moments at the edge of the slab and therefore may induce torsional and bending moments into the beam.[8,9]

REFERENCES

8. Saeger, W., "Uber die Berücksichtigung der Torsionssteifigkeit der Randbalken von Stahlbetondecken," *Beton- und Stahlbetonbau* (Berlin), V. 45, No. 10, 1950, 230-235.

9. Lindner, H., "Über die Einspannung von Stahlbetonfahrbahnplatten in drillsteife Randtraeger," *Beton- und Stahlbetonbau* (Berlin), 1955, pp. 264-267.

10. Müller, L. S., "Restraint of Slabs by Edge-Beams," *Concrete and Constructional Engineering* (London), V. 53, No. 7, 1958, pp. 251-263.

11. Leistner, H., "Nochmals torsionselastische Einspannung," *Die Bautechnik* (Berlin), 1958, pp. 140-142.

12. Cakiroglu, A., "Über die auf torsionssteife Randtraeger aufliegende Rippendecke," *Der Bauingenieur* (Berlin), 1958, pp. 294-299.

By AMIN GHALI*

The author has tackled successfully one of the problems which has direct application in the daily work of a design office. Design by the author's method leads to a considerable reduction of concrete, and reinforcement which is extended into the panels of the adjoining cantilever slabs.

In the derivation of the general differential equation an elementary strip was considered (Fig. B). The ends of the beam were assumed rigidly fixed, i.e., no rotation takes place at the ends. It follows from this assumption that there will be no deflection in the end strips along the lines AD and BC. Hence we may consider the slab simply supported on the three sides AD, DC, and BC.

The relation between the rotation ϕ and the moment m gives the stiffness of an elementary strip (Author's Eq. 4)

$$\phi = -\frac{m\,dx\,S}{3EI} \qquad (4)$$

The stiffness or the moment which makes a unit rotation in a strip of unit width is

$$\eta = -\frac{m}{\phi} = \frac{3EI}{S}$$

The stiffness of a slab is considerably more than the stiffness of separate strips. The bending of the slab in one direction (see Line 1-1 of Fig. B) is accompanied by a bending in the perpendicular direction. The value of η will be more than the above value, and will depend

*Lecturer in Structural Engineering, Cairo University, Cairo, Egypt.

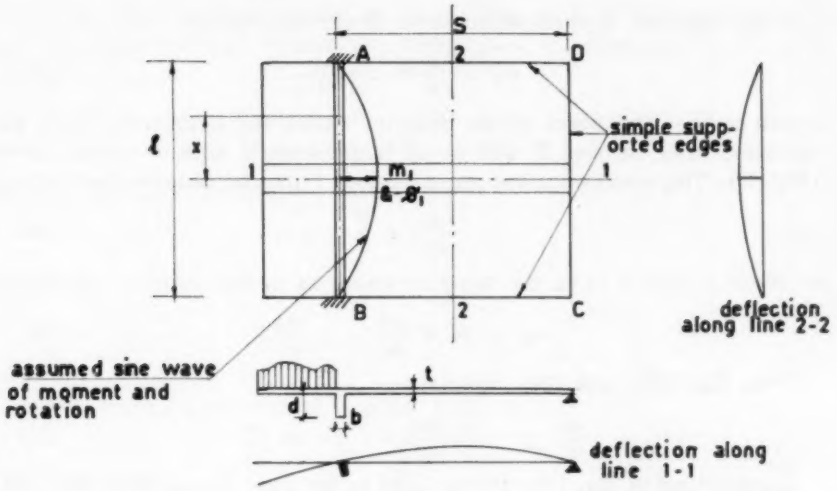


Fig. B

upon the length of the slab sides l and s. To simplify the problem assume the moment m to be distributed along the edge AB (Fig. B) according to a cosine curve such that

$$m = m_1 \cos \frac{\pi x}{l} \qquad (24)$$

It follows that the rotation varies in a similar way

$$\phi = \phi_1 \cos \frac{\pi x}{l} \qquad (25)$$

m_1, ϕ_1, are the moment and the rotation at the center of side AB.

From the above relations we have

$$\frac{m}{\phi} = \frac{m_1}{\phi_1} = -\eta \qquad (26)$$

The value of η here represents the bending stiffness of the edge AB of the slab. η may be expressed in terms of a numerical coefficient C_k which is a function of the shape of the slab, the quantity $D = Et^3/12(1-\mu^2)$,* and the length of the span S as follows

$$\eta = C_k \frac{D}{S} \qquad (27)$$

The values of C_k for different slab ratio S/l were calculated by N. M. Newmark[13] when the end DC is simply supported and totally fixed. Some of Newmark values are given in Table A.

* μ is Poisson's ratio. The quantity $Et^3/12(1-\mu^2)$ better replaces EI as a measure of the flexural rigidity of the slab.

If the external applied moment m_o is substituted by

$$m_o' = \frac{4}{\pi} m_o \cos \frac{\pi x}{l} \quad \text{(28)}$$

which is the first term of the Fourier series for a uniform load, the variation of ϕ, m, and T will be of trigonometric sine or cosine curve (Fig. C). The maximum values of m and T can be obtained as follows

$$\frac{d\phi}{dx} = \frac{T}{C} \quad \text{(29)}$$

in which C and T have the same meaning as in the authors' equations.

$$m_o' - m = \frac{dT}{dx} = C \frac{d^2\phi}{dx^2} \quad \text{(30)}$$

From Eq. (26) and (24), we have

$$\frac{d\phi}{dx} = -\frac{1}{\eta} \frac{dm}{dx} = \frac{m_1 \pi}{\eta l} \sin \frac{\pi x}{l} \quad \text{(31)}$$

Substituting in Eq. (30) for m_o' and m by their values from Eq. (24) and (28), we get

$$\frac{4}{\pi} m_o \cos \frac{\pi x}{l} - m_1 \cos \frac{\pi x}{l} = \frac{C m_1 \pi^2}{\eta l^2} \cos \frac{\pi x}{l}$$

$$m_1 = \frac{4}{\pi} \cdot \frac{m_o}{1 + \frac{C}{\eta} \frac{\pi^2}{l^2}} \quad \text{(32)}$$

or

$$m_1 = m_{max} = \frac{4}{\pi} \frac{m_o}{(1 + \alpha)} \quad \text{(33)}$$

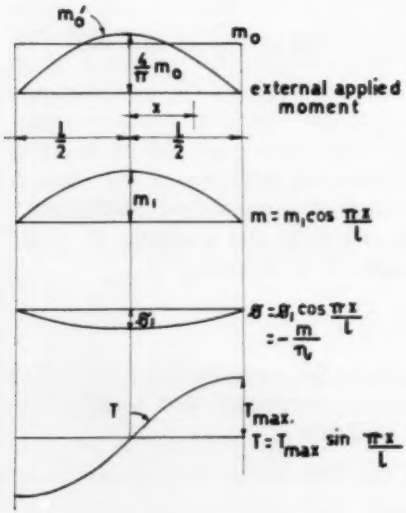


Fig. C—Variation of m_o', m, ϕ, and T along the edge AB of the slab

α is a dimensionless coefficient depending on b, d, t, s, and l which are the beam and slab dimensions. Integrating the first part of Eq. (30), we get

$$T = \frac{4l}{\pi^2} m_o \sin\frac{\pi x}{l} - \frac{l}{\pi} m_1 \sin\frac{\pi x}{l} + \text{constant}$$

at $x = 0$, $T = 0$, therefore constant $= 0$

$$\therefore T = \frac{l}{\pi}\left(\frac{4}{\pi} m_o - m_1\right) \sin\frac{\pi x}{l} \qquad (34)$$

at $x = l/2$

$$T_{max} = \frac{l}{\pi}\left(\frac{4}{\pi} m_o - m_1\right)$$

substituting for m_1 by its value in Eq. (33), we get

$$T_{max} = \frac{4l}{\pi^2} m_o \left(\frac{\alpha}{1+\alpha}\right) \qquad (35)$$

A more accurate value of T_{max}, may be obtained if the original constant distribution of m_o is considered.

Eq. (30) will then be

$$\frac{dT}{dx} = m_o - m$$

$$\int_0^{l/2} dT = \int_0^{l/2} m_o\, dx - \int_0^{l/2} m dx \qquad (35a)$$

$$T_{max} = m_o \frac{l}{2} - m_1 \frac{l}{\pi}$$

$$T_{max} = m_o \frac{l}{2} - \frac{4l}{\pi^2} \frac{m_o}{(1+\alpha)}$$

$$T_{max} = \frac{m_o l}{2}\left(\frac{\alpha + 0.19}{\alpha + 1}\right) \qquad (36)$$

In order to compare the results obtained with the above equations and those given by the author, we solve Example 1 and 2 by considering the values of C and η the same as he did.

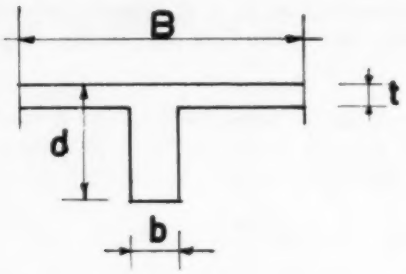
Example 1

$$\eta = \frac{3EI}{S} = \frac{Et^3}{4S}, \quad \alpha = \frac{C}{\eta} \cdot \frac{\pi^2}{l^2} = \frac{4CS}{Et^3} \cdot \frac{\pi^2}{l^2}$$

putting

$$\frac{Et^3}{4CS} = 1 \text{ and } l = 1; \quad \alpha = \pi^2 = 9.87$$

Fig. D



and

$$m_{max} = \frac{4}{\pi} \frac{m_o}{(1 + 9.87)} = 0.117\ m_o$$

$$T_{max} = \frac{m_o}{2}\left(\frac{9.87 + 0.19}{9.87 + 1}\right) = 0.463\ m_o$$

The values obtained by the author are

$$m_{max} = 0.114\ m_o \text{ and } T_{max} = 0.463\ m_o$$

Example 2

Given: $b = 10$ in., $d = 20$ in., $l = 255$ in., $t = 4$ in., $S = 144$ in., and applied moment $= m_o$

$$\frac{d}{b} = 2, C = 0.229\ G \times 10^2 \times 20 = 4580\ G \text{ and } \eta = \frac{Et^3}{4S} = \frac{E \times 4^3}{4 \times 144} = \frac{E}{9}$$

We take $G = 3/7\ E$ (the same as considered by the author in the calculation of Table 2).

$$\alpha = \frac{C}{\eta} \frac{\pi^2}{l^2} = \frac{4580}{1/9} \times \frac{3}{7} \cdot \frac{\pi^2}{255 \times 255} = 2.7$$

$$m_{max} = \frac{4}{\pi} \frac{m_o}{(1 + 2.7)} = 0.34\ m_o$$

and

$$T_{max} = \frac{m_o\, l}{2}\left(\frac{2.7 + 0.19}{2.7 + 1}\right) = 0.39\ m_o\, l$$

The values obtained by the author are

$$m_{max} = 0.33\ m_o \text{ and } T_{max} = 0.4\ m_o\, l$$

If in the above example the bending stiffness of the slab is taken from Table A, the bending moment m resisted by the slab will be more than that calculated above while the torsional moment T will be decreased.

When calculating the torsional rigidity of the Beam C, a certain width B of the slab must be taken (Fig. D) as acting with the web, thus

forming a T section. Experimental work is still needed to determine more precisely what this width should be. One of the numerous values of B recommended by various authors is $B = 11\ t + b$.

The torsional stiffness of a T section is given by several authors.[14]

It may be concluded that the suggested values of the torsional stiffness C of the beam and the bending stiffness of the slab may be substituted either in the formulas obtained by the author or those given above.

The solution given above may be easily extended to take account of the deflection of the beam which will vary as a sine wave.

TABLE A — NUMERICAL COEFFICIENT C_k FOR THE STIFFNESS OF A PANEL OF SLAB

Ratio of sides S/l	C_k — when far end DC is simply supported	C_k — when far end DC is totally fixed
0	3.0	4.0
0.2	3.16	4.11
0.4	3.62	4.43
0.6	4.36	4.97
0.8	5.31	5.72
1.0	6.41	6.65
1.2	7.59	7.72
1.4	8.82	8.88
1.6	10.06	10.09
1.8	11.31	11.32
2.0	12.57	12.57
2.5	15.71	15.71
3.0	18.85	18.85
3.5	21.99*	21.99

REFERENCES

13. Newmark, Nathan M., "A Distribution Procedure for the Analysis of Slabs Continuous Over Flexible Beams," *Bulletin* No. 84, University of Illinois, 1938.

14. Peabody, Dean, *The Design of Reinforced Concrete Structures*, J. Wiley and Sons, New York, 2nd Edition, pp. 416 and 417.

By H. KLASMER†

Mr. Gouda's paper solves some interesting new details of the problem which has been treated before by W. Saeger[8] and recently by L. S. Muller.[10]

As I pointed out in a remark on Mr. Muller's paper‡ all these solutions neglect the effect of the monolithic connection between the "elementary strips" of the slab. Mr. Gouda too uses this same simplification which in my opinion is not justified. For there are bending moments acting in the slab in the direction parallel to the edge beam as well as torsional moments between the elementary strips and their total effect will have a similar order of magnitude as that of the torque of the edge beam alone.

---

*Note: for values of S/l greater than 3.4, the value of C_k is the same for both cases considabove and equal to

$$C_k = 2\pi \frac{S}{l} = 6.28 \frac{S}{l}$$

†Civil Engineer, Tel Aviv, Israel.

‡*Concrete and Constructional Engineering* (London), V. 54, No. 6, June 1959.

Mr. Gouda's and Mr. Muller's results are certainly applicable to a "concrete joist floor" (or "ribbed slab") spanning in one direction without any distribution rib. But where there is a distribution rib of the usual size it tends to equalize the deflections of the main ribs by exerting a concentrated force on each rib, acting downwardly on the ribs of the column strip and upwardly on those of the middle strip. An analogous effect exists in a monolithic (full concrete) slab and may be even more marked.

As early as 1946 I published a paper on the anchoring of a cantilever slab by the combined action of the edge-beam and an internal floor slab with supports on four sides [*Engineering Forum*, Tel Aviv, (in Hebrew)], and another paper on the fixing of such an internal slab by the torque of an elastic edge beam [*Journal of the Engineers' and Architects' Association* (Israel), Mar. 1953] with an abstract in English (see also *Engineering Index*, 1953). Just now I am finishing for publication a paper which contains the investigation and solution for a concrete joist floor with one or two distribution ribs which is restrained by elastic edge beams on elastic columns, with or without cantilever slabs.

The results obtained for this latter type of structure may be suitable for further research so as to find a more reliable analysis of the monolithic slab spanning in one direction and restrained by an elastic edge beam.

By THOMAS PAULAY*

The discusser studied the author's theoretical investigations on the interaction of beams in torsion and slabs in bending with great interest. The author is to be congratulated for treating this neglected subject. An extension of his investigation to cover the case of two-way slabs as well, would have been most welcomed.

The writer carried out similar investigations in which it was attempted to arrive at sufficiently accurate results using the method of bending moment distribution of Hardy Cross. The first object was to obtain the basic stiffness values for the various members subject to bending or torsion. To do this, the simplified conditions analyzed by the author were studied and the results were in full agreement with those shown by the author.

The general condition more frequently met in design practice is indicated in Fig. E. To apply the author's investigations in combination with the successive approximation of moment distribution, the following approach may be used.

*Member American Concrete Institute, D. Bruce-Smith & Associates, Consulting Engineers, Wellington, New Zealand.

ROTATION OF THE EDGE BEAM

It is assumed that the edge beam between Columns A-A is infinitely stiff. The distribution of end fixing moments for the slab m_A will be uniform Fig. F1. If the torsional stiffness of the Beam A-A is of finite value, comparable with the flexural stiffness of the slab, rotations will occur thereby relaxing moments at the edge of the slab. As the beam is fixed against rotation at its supports A-A, it can be seen that the distribution of edge moments after relaxation of the beam will be of the pattern shown in Fig. F2 and Fig. 13. The smallest restraining moment m_o will occur at midspan where rotations will be largest relative to the fixed supports of the beam. The magnitude of m_o will depend on the relative stiffness of the slab in flexure and the beam in torsion. The ratio

$$\frac{m_o}{m_A} = \mu \qquad \text{.....(37)}$$

may be called a distribution factor applicable to the beam at the center 0 of the slab.

Considering any strip of unit width across the beam and slab as shown in Fig. G under the uniformly distributed load g and following the steps carried out by the author in Eq. (12) to Eq. (15) the following results are obtained:

$$m(x) = m_A \frac{\cosh fx}{\cosh f\frac{l}{2}} \qquad \text{.....(38)}$$

where

$$f^2 = \frac{4EI}{SC} \text{ and } I = \frac{t^3}{12} \qquad \text{.....(38a)}$$

The torsion at any distance x from the center of the beam is

$$T(x) = \int_0^x m(x)dx = \frac{m_A \sinh fx}{f \cosh f\frac{l}{2}}$$

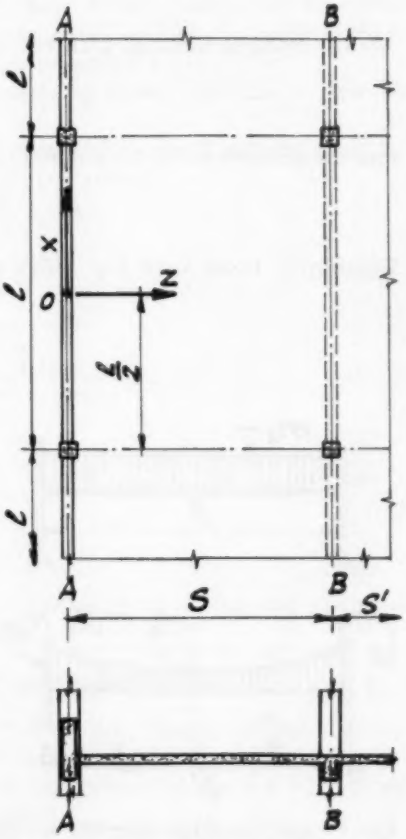


Fig. E—One-way slab and torsion resistant beams

The maximum torque at the supports A of the beam is obtained by putting $x = l/2$

$$T_A = \frac{m_A}{f} \tanh f\frac{l}{2} \quad \text{(39)}$$

The minimum value of edge moment at the center of the beam is obtained by putting $x = 0$ from Eq. (38)

$$m_o = \frac{m_A}{\cosh f\frac{l}{2}}$$

and substituting into Eq. (37)

$$\mu = \frac{1}{\cosh f\frac{l}{2}} \quad \text{(40)}$$

Let the relative flexural stiffness of the center strip of the slab be

$$k_s = \frac{EI}{S} = \frac{Et^3}{12S} \quad \text{(41)}$$

and the relative torsional stiffness of the marginal beam be

$$K_T = \frac{C}{l^2} = \frac{\beta b^3 dG}{l^2} \quad \text{(42)}$$

Combining these with Eq. (38a) and Eq. (40) it is found that

$$\mu = \frac{1}{\cosh\left(\frac{k_s}{K_T}\right)^{1/2}} \quad \text{(43)}$$

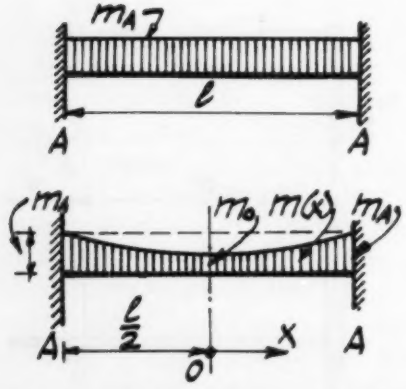


Fig. F1 and F2—Edge moments before (above) and after (below) relaxation of beams

The value of the distribution factor may be obtained for convenience from Fig. H.

By combining Eq. (43) and Eq. (39) the maximum torque can be obtained thusly

$$T_A = \frac{m_A l}{\omega} \quad \text{(44)}$$

where

$$\omega = \frac{2\sqrt{k_s/K_T}}{\tanh\sqrt{k_s/K_T}}$$

the values of which are again shown for convenient use in Fig. H. If m_A and m_o (Fig. F2) are given the value of μ can be determined

by Eq. (24), and the corresponding value of ω can be located on Fig. H.

Any applied (or out of balance) moment can now be distributed between the marginal beam and the slab. The distribution refers strictly to the center strip of the slab. Naturally, balancing quantities must be carried over to the opposite supports of the slab in the usual way. Similar procedure is followed at interior beams where two slabs, of possibly different stiffnesses, join the beam. The distribution factors necessary to carry out the moment distribution can be found as follows (see Fig. I).

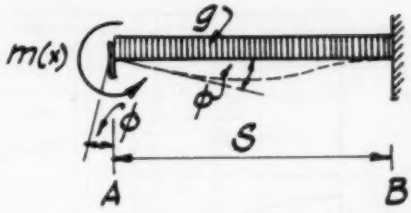


Fig. G—Load on unit strip of slab

1. Consider Slab A and Beam B only. Obtain the stiffness ratio k_s/K_T and from Fig. H the appropriate μ value. Let this distribution factor of the beam in respect of Slab A be called μ_A.

2. Consider Slab C and Beam B only and repeat the above process obtaining μ_C.

3. By simple proportion it can be shown that the final distribution factors for the three members meeting at B are—

for Slab A

$$\nu_A = \frac{(1 - \mu_A)\mu_C}{\mu_A + \mu_C - \mu_A \mu_C} \quad \text{(45a)}$$

for Slab C

$$\nu_C = \frac{(1 - \mu_C)\mu_A}{\mu_A + \mu_C - \mu_A \mu_C} \quad \text{(45b)}$$

for the Beam B

$$\nu_B = \frac{\mu_A \mu_C}{\mu_A + \mu_C - \mu_A \mu_C} \quad \text{(45c)}$$

Naturally

$$\nu_A + \nu_B + \nu_C = 1 \quad \text{(45d)}$$

The balancing of moments along the center strip of the slab can now by carried out for any number of bays.

ROTATION OF COLUMNS

After the equilibrium conditions for slabs and beams has been established, the effect of accumulated edge moments on the columns must be considered. So far, it was assumed that supporting beams are fully fixed against torsional rotation at their supports.

The 'out of balance' moments, which result at the supports of the beams from edge moments, must be absorbed by the columns and slabs

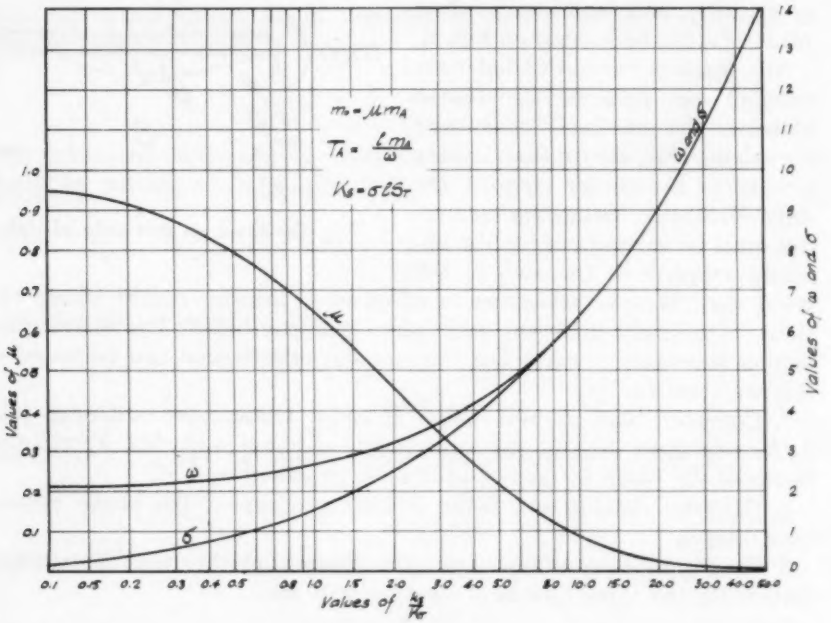


Fig. H—Relationship of the coefficients μ, ω, and σ to stiffness ratio K_B/K_T

meeting at the joint. The stiffness of the whole of the slabs which are connected to the columns by torsion-resistant beams must be considered in order to compare their flexural rigidity with that of the columns. The following convenient definition may be made:

> "The flexural stiffness of a one-way slab, which is framing into a beam at one end and is fixed against rotation at the opposite end, is that torsional moment which, when applied at the supports of the edge beam, causes unit rotation there."

As a result of the applied moment T_A at the supports of the edge beam (Fig. J) let the rotation there be λ. The corresponding moment for a unit strip of the slab will be (Fig. G)

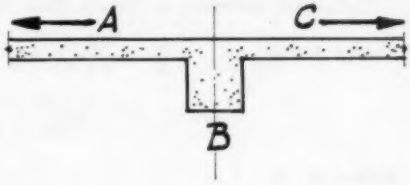
$$m_A = \frac{4EI}{S} \lambda$$

Substituting this expression into Eq. (39) and putting $\lambda = 1$ according to the definition

$$T_A = \frac{4EI}{Sf} \tanh f\frac{l}{2} = \text{stiffness of the slab}$$

Using the stiffness ratios as shown previously, the simple working equation for the stiffness of a 'one-way' slab of $l/2$ width is obtained thus

Fig. I — Distribution of moments between slabs and beams



$$K_s = \sigma l K_T \qquad (46)$$

where

$$\sigma = 2\sqrt{\frac{k_s}{K_T}} \tanh \sqrt{\frac{k_s}{K_T}}$$

the values of which are shown in Fig. H.

FINAL MOMENTS

1. In the first part of this discussion it was shown how moments at the ends of the center strip of the slab are obtained assuming that all beams are fixed against rotation at their supports (value of μ from Fig. H).

2. The summation of slab edge moments along the beams, Eq. 44, gives the fixed end torque at the supports of the beams.

3. An equal and opposite moment to above torque is applied at the joint and is distributed between columns and slabs, through the beams, according to their defined stiffnesses which are derived at from Eq. 46.

4. Results from steps 1 and 3 are added and final moments obtained. Note, that once the value of μ for a beam is found, m_A and m_o can also be found from the torque which resulted from balancing moments at the slab-column joints (step 3).

EXAMPLE

The floor framing shown in Fig. E is as follows:

l = 20 ft	Beam A—A 48 x 10 in.
S = 16 ft (Slab A—B)	Beam B—B 20 x 14 in.
S' = 10 ft (Slab B—C)	Column A 14 x 18 in.
All slabs 6 in. thick	Column B 16 x 16 in.
Load g = 180 lb per sq ft	Floor heights 10 ft-6 in. (intermediate identical floors)

The floor framing is symmetrical about the centerline of Slab B—C.

Stiffnesses

Slab stiffness of middle strips calculated from Eq. (41) omitting E

Slab A—B $\qquad K_s = \dfrac{6^3}{12 \times 16 \times 12} = 0.094$ sq in.

Slab B—C $\qquad K_s = \dfrac{6^3}{12 \times 10 \times 12} = 0.150$ sq in.

Torsional stiffness of beams from Eq. (42)

Beam A—A $\quad \frac{d}{b} = \frac{48}{10} = 4.8 \therefore \beta = 0.29 \frac{G}{E} \approx 0.43$

$$K_T = \frac{0.29 \times 10^3 \times 48 \times 0.43}{(20 \times 12)^2} = 0.104 \text{ sq in.}$$

Beam B—B $\quad \frac{d}{b} = \frac{20}{14} = 1.43 \therefore \beta = 0.19$

$$K_T = \frac{0.19 \times 14^3 \times 20 \times 0.43}{(20 \times 12)^2} = 0.078 \text{ sq in.}$$

Flexural stiffness of columns

Assume points of contraflexure halfway between floors. Therefore

$$K_{ca} = \frac{3\,EI}{(H/2)}$$

Column A $\quad K_A = \frac{3 \times 18 \times 14^3}{12 \times 63} = 196 \text{ cu in.}$

Column B $\quad K_B = \frac{3 \times 16^4}{12 \times 63} = 260 \text{ cu in.}$

Flexural stiffness of slab panels of $l/2 = 10$ ft width

Panel A—B at A $\quad \frac{K_s}{K_T} = \frac{0.094}{0.104} = 0.90$ from Fig. H $\sigma = 1.40$

$\therefore$ from Eq. (46) $\quad K_s = 1.40 \times 20 \times 12 \times 0.104 = 35.0$ cu in.

Panel A—B at B $\quad \frac{K_s}{K_T} = \frac{0.094}{0.078} = 1.20 \quad \therefore \sigma = 1.70$

and $\quad K_s = 1.70 \times 20 \times 12 \times 0.078 = 31.8$ cu in.

Panel B—C at B or C $\quad \frac{K_s}{K_T} = \frac{0.150}{0.078} = 1.92 \quad \therefore \sigma = 2.50$

and $\quad K_s = 2.50 \times 20 \times 12 \times 0.078 = 46.8$ cu in.

Distribution Factors

Slab middle strips

at A $\quad \frac{K_s}{K_T} = 0.90$ hence from Fig. H and Eq. (37)

for Beam A—A $\quad \mu_A = 0.68$

therefore for Slab A—B $\quad u = \underline{0.32}$

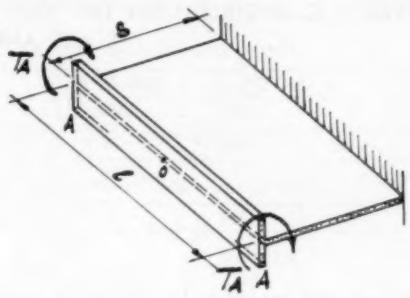
1.00

at B from Fig. H $\quad \mu_A = 0.62$ and $\mu_C = 0.47$

hence from Eq. (45a) $\quad v_A = \frac{(1 - 0.62) \times 0.47}{0.62 + 0.47 - 0.62 \times 0.47} = \frac{0.38 \times 0.47}{0.80} = 0.23$

and from Eq. (45c) $\quad v_B = \frac{0.62 \times 0.47}{0.80} = 0.36$

Fig. J—Torsional moments applied to edge beam defining flexural stiffness of slab



and from Eq. (45b) $$v_c = \frac{(1 - 0.47) \times 0.62}{0.80} = 0.41$$

check from Eq. (45d) $$\Sigma = 1.00$$

Column and slabs — (Note that column above and below are identical and slabs on either side of a column are also identical — hence one column and one slab may be used.)

at A

$$K_{Column} = 196 \times 0.850$$

$$K_{Slab\ A-B} = \frac{35 \times 0.150}{231 \times 1.000}$$

at B

$$K_{Slab\ B-A} = 31.8 \times 0.094$$

$$K_{Column} = 260.0 \times 0.768$$

$$K_{Slab\ B-C} = \frac{46.8 \times 0.138}{338.6 \quad 1.000}$$

Fixed end moments of slab middle strips

Slab A—B at A and B $= 0.180 \times 16^2 \times 12/12 = \mp 46.2$ in.-kip per ft

Slab B—C at B and C $= 0.180 \times 10^2 \times 12/12 = \mp 18.0$ in.-kip per ft

TABLE B—DISTRIBUTION OF SLAB MIDDLE STRIP MOMENTS (IN.-KIP PER FT)

Beam A	Slab A—B		Slab B—A	Beam B	Slab B—C		
0.68*	0.32*		0.23*	0.36*	0.41*		
—	−46.2		+46.2	—	−18.0	(+28.2)	Symmetry
+31.4	+14.8		− 6.5	−10.2	−11.6		
—	− 3.3		+ 7.4	—	+ 5.8	(+13.2)	
+ 2.2	+ 1.1		− 3.0	− 4.8	− 5.4		
—	− 1.5		+ 0.5	—	+ 2.7	(+3.2)	
+ 1.0	+ 0.5		− 0.7	− 1.1	− 1.3		
+34.6	−34.6		+43.9	−16.1	−27.8		

*Distribution factors arrived at above.

TABLE C—DISTRIBUTION OF "OUT OF BALANCE" MOMENTS BETWEEN SLABS AND COLUMNS

	Col. A	Slab A—B		Slab B—A	Col. B	Slab B—C	
	0.850	0.150		0.094	0.768	0.138	—Symmetry—
−385	+327	+58	+198	−19	−152	−27	
	—	−10		+29	—	+13	
	+ 9	+ 1		− 4	− 32	− 6	
−385	+336	+49	+198	+ 6	−184	−20	

Fixed end moments for beams in torsion

Beam A $\mu = \frac{34.6}{46.2}$ from Eq. (37) $= 0.75$

therefore from Fig. D $\omega = 2.4$

and from Eq. (44) $T_A = -\frac{20 \times 46.2}{2.4} = -385$ in.-kip

Beam B $\mu = \frac{16.1}{28.2} = 0.57 \therefore \omega = 2.85$

and $T_B = \frac{20 \times 28.2}{2.85} = 198$ in.-kips

Final moments

Torsion in Beam A—A

from 1st distribution (D1) $=$ − 385 in.-kip

from 2nd distribution (D2) $=$ + 49 in.-kip

T_A − 336 in.-kip

Column moment in Column A

from (D1) $=$ ———

from (D2) $=$ + 336 in.-kip

$M_{col} =$ + 336 in.-kip

Column strip moment of Slab at A

from (D1) $= -46.2$ in.-kip per ft

$\mu = 0.68$ (See distribution factors)

hence $\omega = 2.6$ thus using Eq. (44)

from (D2) $M_A = \frac{\omega T_A}{l} = \frac{2.6 \times 49}{20} =$ + 6.0 in.-kip per ft

$M_A =$ − 40.2 in.-kip per ft

Middle strip moment of Slab at A

from (D1) $= -$ 34.6 in.-kip per ft

using Eq. (37) from (D2) above 0.68×6.0 $= +$ 4.1 in.-kip per ft

$M_o = -$ 30.5 in.-kip per ft

Check torsion in Beam A—A from edge moments of Slab A—B at A

Eq. (37) $\frac{M_o}{M_A} = \frac{30.5}{40.2} = 0.76$ $\therefore$ Fig. D $\omega = 2.38$

and from Eq. (44) $T_A = \frac{20 \times (-40.2)}{2.38} = -338 \simeq -336$ in.-kip

Column strip moment of Slab B—A at B

from (D1) $= +$ 46.2 in.-kip per ft

$\mu = 0.62$ $\therefore$ $\omega = 2.80$

from (D2) $\frac{2.80 \times 6.0}{20} = +$ 0.8 in.-kip per ft

$M_B = +$ 47.0 in.-kip per ft

Middle strip moment of Slab B—A at B

from (D1) $= +$ 43.9 in.-kip per ft

from (D2) $0.62 \times 0.8 = +$ 0.5 in.-kip per ft

$M_o = +$ 44.4 in.-kip per ft

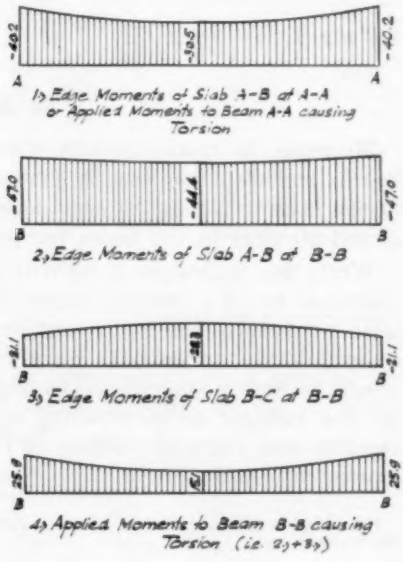


Fig. K—Moment Distribution diagrams for flat slab

Column strip moment of Slab B—C at B or C

from (D1) $= -$ 18.0 in.-kip per ft

$\mu = 0.47 \therefore \omega = 3.15$

from (D2) $\dfrac{-20 \times 3.15}{20} = -$ 3.1 in.-kip per ft

$M_a = -$ 21.1 in.-kip per ft

Middle strip moment of Slab B—C at B or C

from (D1) $= -$ 27.8 in.-kip per ft

from (D2) $-3.1 \times 0.47 = -$ 1.5 in.-kip per ft

$M_o = -$ 29.3 in.-kip per ft

Torsion in Beam B—B

$\Sigma M_a = 47.0 - 21.1 = 25.9$ in.-kip per ft

$\Sigma M_o = 44.4 - 29.3 = 15.1$ in.-kip per ft

Therefore $\mu = \dfrac{15.1}{25.9} = 0.58$ and $\omega = 2.85$

from Eq. (44) $T_s = \dfrac{20 \times 25.9}{2.85} = 182$ in.-kip

Column moment in Column B

from (D1) $=$ ———

from (D2) $= -184$ in.-kip

$M_{col} = -184$ in.-kip

Check: $T_s = 182$ in.-kip, $M_{col} = -184$ in.-kip } approx. equal

See Fig. K

By A. SIEV*

Moments in monolithically connected reinforced concrete marginal beams and slabs are usually provided for in principle by bending some of the reinforcing bars so as to resist the negative moments, and using closed stirrups in the beam for torsional shear stresses.[15,16]

While this technique is evidently satisfactory with regard to negative moments (with a varying factor of safety), it is not necessarily the case with torsional moments. A *simple* method for deriving these moments quantitatively (even with an error of say, no more than 10 percent) would be highly appreciated by engineers. *However, the error introduced by the concept of elementary strips, neglecting the plate's torsional rigidity and flexural rigidity in the* x *direction, is not clear and may exceed the reasonable limit.*

*Member American Concrete Institute, Lecturer, Technion—Israel Institute of Technology, Haifa, Israel.

Fig. L—Torsional rigidity produced by T-beam

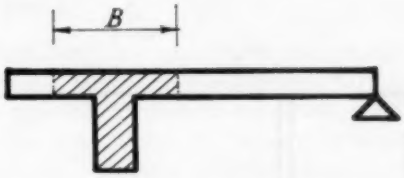


Fig. L (based on Fig. 1) shows that the monolithical beam-slab system acts as a T-beam with regard to torsional rigidity and has in this case a width B (unknown) for the effective flange. It is clear that the larger the plate dimensions (S, s, and t) as compared to those of the beam's cross section (b and d), the more pronounced will be the effect of neglecting the plate's torsional rigidity.

The same effect can also be considered from another point of view. Fig. Mb shows that there is an upward deflection of the inner slab and that its maximum is at point A ($y = 0.573\,S$) being

$$\delta = \frac{2}{3} \cdot \frac{\phi}{2} \cdot 0.573S = 0.191\,\phi\,S \qquad (47)$$

Fig. Mc shows a section through A with transverse plate deformation δ. The corresponding flexural moments, on the line $y = 0.573\,S$, in the plate will be:

$$M_{x,\,plate} = EI\,\frac{d^2\delta}{dx^2} = 0.191\,EIS\,\frac{d^2\phi}{dx^2} \qquad (48)$$

Differentiating Eq. 2 and introducing the value of dT/dx from Eq. 7a we get:

$$\frac{d^2\phi}{dx^2} = \frac{m_o}{C}\,\frac{\cosh\,wx}{\cosh\,\frac{wl}{2}} \qquad (49)$$

or, introducing into Eq. (48)

$$M_{x,\,plate} = \frac{0.191\,EIS}{C}\,m_o\,\frac{\cosh\,wx}{\cosh\,\frac{wl}{2}} = 0.191\,u^2\,\frac{S^2}{3l^2}\,m_o\,\frac{\cosh\,wx}{\cosh\,\frac{wl}{2}} \qquad (50)$$

Example*

A 10 x 20 in. beam with span $l = 255$ in. is subjected to a cantilever moment m_o on one side and supports an inner slab of $t = 8$ in. and span 1000 in. on the other side. The beam is supposed to be suspended by sufficiently close-spaced cables throughout its span, so as to avoid deflection while permitting free rotation.

$$u = \frac{K\,t\,l}{b^2}\left(\frac{t}{S}\right)^{1/2} = 2.07$$

*Deviations from Example 2 are deliberate in order to accentuate the error involved.

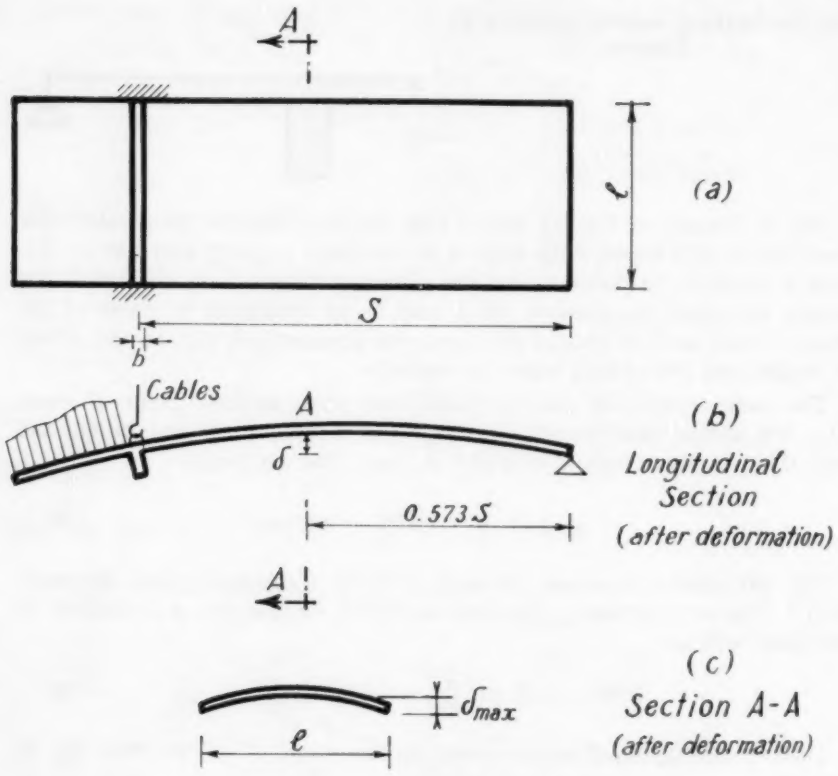


Fig. M—Two-way-bending of plate

$M_{x,\,plate}$ will be calculated for $x = l/2$, where it reaches its maximum.

$$M_{x,\,plate} = 0.191 \cdot 2.07^2 \cdot \frac{1000^2\, m_o}{3 \cdot 255^2} = 4.23\ m_o$$

It is seen that the "secondary" moment is of higher order than the primary moment (apart from the fact that its maximum would be at the edge, with no external moment as counterbalance). This is intended to suggest the weakness of the author's theory in general.

REFERENCES

15. ACI Committee 315, *Manual of Standard Practice for Detailing Reinforced Concrete Structures*, (ACI 315-57)," American Concrete Institute, 1957.

16. Reynolds, Charles E., *Reinforced Concrete Designer's Handbook*, Concrete Publications Ltd., London, p. 167, Fig. 25 b.

By M. YEHYA*

The assumptions of homogeneity and elasticity on which the mathematical solution of this problem is based is in my opinion not totally satisfactory. I say this because there are *two* different materials used, namely the reinforcing steel and the concrete, which act as the resisting element. In such a case it is preferable to introduce a factor or factors to represent any variation of behavior which may occur due to the heterogeneity of the resisting material, the percentage of the reinforcing steel and its effect on increasing the rigidity of a part of the section, or any other factor.

AUTHOR'S CLOSURE

The writer is deeply indebted to Mr. T. Paulay for adding his interesting material (used Hardy-Cross-moment distribution method) to the study on continuous slabs. As requested by Mr. Paulay, the writer hopes in the near future to extend his paper to cover the case of the two way slab.

Prof. M. Yehia calls for introducing a factor to cover the assumptions of homogeneity and elasticity on which the solution is based. In fact such a factor could only be determined by carrying out some experimental tests, which the writer intends to do. But the writer thinks that under working loads, based on his own experience from other experimental works, the variation between the theoretical and the experimental values will be small and therefore may be neglected for the simplicity of the problem.

It is regrettable that Mr. Siev can not appreciate the method of analysis presented in the paper. He bases his discussion against the writer's concept which apparently neglects the plate's torsional rigidity and flexural rigidity in the x direction, but he should bear in mind that the slab is an one way slab and in the s direction.

In such a slab the reinforcement in the x direction will be the only distributors, besides the twisting moment actually is very small in this case for this kind of slab and test results have shown that such slabs are incapable of resisting twisting moments.

The example which he gives has no practical significance and as a matter of fact, is quite misleading.

The approach to the problem given by Dr. A. Ghaly is a worthy one. He points out that the stiffness of the whole slab is more than the stiffness of separate strips. This fact in general is true, but the writer assumes strips, where the slab is one way, in order to arrive at a simple and practical solution for the problem. Moreover the difference be-

*Member American Concrete Institute, Assistant Professor, Faculty of Engineering, Cairo University, Giza, Egypt.

TABLE D — TORSIONAL RIGIDITY

Type of section	J, in.^{4}		
	$d/b = 2$	$d/b = 3$	$d/b = 4$
T	5,800	9,652	12,550
Rectangular	4,580	7,892	11,240
Difference, percent	25	18	10

tween the two stiffnesses is small as is shown by use of Newmark's table,[14] in the following:

Assuming a slab of separate strips $\therefore C_k = 3$

Assuming the whole slab where $\frac{s}{l} = 0 \rightarrow 0.5$

corresponding $C_k = 3 \rightarrow 4$

This means that the difference between slab stiffnesses for both assumptions in the extreme case, where $s/l = 0.5$, is 25 percent only.

Dr. Ghali also adds that in calculating the torsional rigidity of the beam, the T-section should be used. Again the writer finds that a simple and practical solution for the problem could be arrived at by choosing a rectangular cross-section in calculating the torsional rigidity of the beam, also the difference in stiffnesses in ordinary cases is small as is shown in the following.

Consider the case where the slab thickness $t = 4$ in. and beam breadth 10 in., and $B = 48$ in.

Torsional rigidity of the rectangular cross-section of the beam $= GJ = G\beta b^3 d$. Torsional rigidity of T section as suggested by Trayer and March $= GJ$ where

$$J = \mu_f \frac{Bt^3}{16} + \mu_w \frac{(d - t)}{16} b^2 + \alpha D^{4*}$$

As can be seen in Table D the differences in the ordinary cases vary between 10-25 percent.

Hence, in the ordinary cases the increase of flexural stiffness of the slab (by considering the whole slab) is accompanied by almost the same increase of the torsional rigidity of the beam (by considering the T-section of the beam) and thus, the constant w shown in Eq. (7) and (8), will almost be the same whether considering the whole slab and T-section beam or considering strips of slab and a rectangular section beam.

*See Reference 14, p. 416.

Discussion of a paper by Inge Lyse:

Shrinkage and Creep of Concrete*

By WASSIL WELEFF†

With increased application of prestressed concrete as main carrying members of different structures, the importance of investigating all factors relating to the behavior of the materials used in the structure is important, timely, and necessary.

The shrinkage and creep phenomena of concrete have been known for many years. Reference here is made to the pioneering work of Mörsch.‡ Consideration of these factors, however, in the design of reinforced concrete structures has been widely underestimated. A large number of European countries have adopted provisions in their code requirements to take adequate care of these factors. In this country, however, only the temporary recommendations for prestressed concrete have certain provisions for considering the shrinkage and creep both of concrete and steel.§ These provisions were dictated more or less due to the nature of prestressed concrete, requiring adequate recognition of the importance of the stresses caused by shrinkage and creep.

The investigations and recommendations of Professor Lyse in this direction are certainly to be acclaimed.

The objective of the discussion here is mainly to draw the attention of designing engineers in this country to the fact that creep, and particularly the shrinkage of concrete, are factors requiring consideration, not only in the design of prestressed concrete, but furthermore in the design of conventionally reinforced and plain concrete.

Whether the ACI Building Code (ACI 318-56) provisions regarding the minimum shrinkage reinforcement (given for floor and roof slab only) are adequate or not is a question. Certainly more investigations and experimental data will be required before adequate knowledge is accumulated for recommendations to be incorporated in a future revision of the building code. Recommendations of this nature should be simple enough and yet adequate for all possible cases.

---

*ACI JOURNAL, V. 31, No. 8, Feb. 1960 (*Proceedings* V. 56), p. 775. Disc. 56-44 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 32, No. 3, Part 2, Sept. 1960 (*Proceedings* V. 56).

†Member American Concrete Institute, Redlands, Calif.

‡Mörsch, E., *Der Eisenbetonbau—Seine Theory and Anwendung*, K. Wittwer, 1923; and *Statik der Gewölbe and Rahmen*, K. Wittwer, 1947.

§ACI-ASCE Committee 323, "Tentative Recommendations for Prestressed Concrete," ACI JOURNAL, V. 29, No. 7, Jan. 1958 (*Proceedings* V. 54), pp. 545-578.

In connection with Professor Lyse's article, the main factors affecting the shrinkage and creep of concrete are listed as follows:

(1) Amount of cement paste
(2) Physical and rheological properties of the ingredients
(3) Temperature and humidity during cure and exposure
(4) Type and magnitude of the sustained stress in percentage of the ultimate strength of the concrete

Professor Lyse's investigations are mainly connected with the first mentioned item.

As further factors influencing the shrinkage and creep of concrete deserving more thorough investigation, the following are recommended (unless included in some of the above mentioned):

(a) Environmental influences—i.e., temperature cycling and humidity variation during cure and mainly at exposure
(b) Cement qualities
(c) Production methods and service conditions—i.e., steam cured; vibrated or nonvibrated concrete; exposure to high temperature, salt water, ice, gases, oils, etc.
(d) Variations in the magnitude and time of the applied creep stress—i.e., increasing and reduction of this stress during the servicing time of the concrete
(e) Type of reinforcement, sectional variations, etc.—i.e., prestressing steel; mild steel reinforcement; combination of both; symmetrical and unsymmetrical sections and reinforcement; etc.

Discussion of a paper by I. L. Tyler:

Long-Time Study of Cement Performance in Concrete. Chapter 12 – Concrete Exposed to Sea Water and Fresh Water*

JAIME DE LAS CASAS P., HOMER M. HADLEY,
R. F. STRATFULL, LEWIS H. TUTHILL,
CARROL M. WAKEMAN and AUTHOR

By JAIME DE LAS CASAS P.†

Mr. Tyler has given us an important chapter in the Long-Time Study Series. The paper is interesting and instructive and the writer is in accordance with most of its conclusions. However, he would like to call attention to some factors dealing with the importance of portland cement composition, that in his judgement, should be pointed out.

Concrete specialists, including harbor engineers, appear to agree that a good, dense, impervious concrete is the most important factor in obtaining durable concrete structures.[7-9] To get this kind of concrete, apart from the factor of portiand cement composition, it is necessary, principally, to have well graded nonreactive aggregates,[10] low water-cement ratio, careful compactation, and proper curing.[11]

The writer believes that the above statement is valid for any kind of environment, including fresh water and sea water, and should always be emphasized. Air-entrained concrete should be used in severe exposure. From a practical point of view and compared with the other factors involved in the production of durable concrete structures in marine environment, the portland cement composition, even though this is a factor, usually appears not to be the most important.[12]‡

The above point of view appears not to be so far from the partial conclusions of Mr. Tyler:

*ACI JOURNAL, V. 31, No. 9, Mar. 1960 (*Proceedings* V. 56), p. 825. Disc. 56-45 is a part of the copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 32, No. 3, Part 2, Sept. 1960 (*Proceedings* V. 56).

†Member American Concrete Institute, Civil Engineer, Lima, Peru.

‡It must be clearly stated that the writer does not say that the differences among the five ASTM portland cements must be discounted, because these cements are different, specially in the gain in strength with age and also in the heat of hydration; these basic differences give them individual construction applications.

p. 830, Cape Cod Canal (sea water exposure): "Among the non-air-entrained specimens, cement composition appears to be of minor consequence, with the Type IV cements having slightly the best average performance in the two rich mixes."

p. 831, Saugerties, N.Y. (fresh water exposure): "Effects of cement composition were minor or nonexistent."

p. 833, Salt Run, St. Augustine, Fla. (sea water exposure): "Differences among the cements appear to be significant for the first time. Cement 18, the highest in C_3A, shows severe surface softening and ravelling at corners."

p. 834, piles at Corona del Mar, Newport Beach, Calif. (sea water exposure): "Effects of cement composition have not appeared."

The exception to the above statement arises only in one marine environment, this was at Salt Run, St. Augustine, Fla., and only in one cement, the highest in C_3A.

Quoting the author (p. 826): "For the St. Augustine installation, Dawes sand from Trudy, Ga., and Montgomery gravel from Montgomery, Ala., materials commonly available at the construction site, were used to construct the piles. All aggregates were considered to be of excellent quality, though the Montgomery gravel has since come under suspicion as potentially alkali reactive."

It is not the writer's intention to say that the possible reactivity explains completely the bad performance of some of the Salt Run piles, but it may cooperate in some degree—at least the suspicion exists.

Let us consider now the influence of high tricalcium aluminate cements in concrete exposed to sea water. In concrete exposed to fresh water, according to Conclusion 4, p. 834, cement composition was not shown to be of significance.

We would touch on a controversial point if, according with the performance of the specimens reported in the paper, we were to state that cements high in C_3A always show strong indications of poor performance. In fact, Mr. Tyler has written on this matter in an impartial and scientific position.

Not only in the St. Augustine experience but also in technical literature there are many other papers and reports that recommend cements low in C_3A based on the "ample evidence" of good performance of this kind of cement, or conversely, on the poor performance of high C_3A cements. But the evidence does not mean that cements high in C_3A always show poor performance.

For instance, in a recent paper[6] the authors recommended cements low in C_3A for concrete in marine environment. However, in the discussion of that paper Homer M. Hadley[13] remarks that "in structures

made with well graded aggregates and built with high C_3A cements no deterioration occurred after many years (50 years) in sea water," as in the San Pedro, Calif., breakwater. The C_3A content of that cement was about 15 percent. Another example was given by Fluss and Gorman.[9] "The taboo placed on high tricalcium aluminate cement is in complete agreement with the findings of many authorities. Among the notable exceptions, so far reported, are the concrete piers under the San Francisco Ferry Building which after 64 years of service are still in good condition, and the concrete cylinder jackets of Pier 17 of the Port of San Francisco. In order to reconstruct the history of Pier 17 and to determine the reason for the excellent performance of the concrete, a thorough investigation was made. The startling fact was that, apparently for all piers of the Port of San Francisco constructed between 1909 and 1916, only high C_3A cement had been used." The calculated C_3A content of the cement used in these piers ranges from 13.6 to 17.2 percent.

In the writer's opinion this finding is not so startling because he knows that in Peru, in the piers of the Port of El Callao, hundreds of reinforced concrete piles were built more than 20 years ago with a cement having at that time a calculated C_3A content of about 15 percent. The aggregates used were river sand and a nonreactive gravel. The performance is similar to the above examples—no softening or ravelling has occurred.

It is possible that new researches could show that, as far as the portland cement composition is concerned, according to raw materials and/or manufacturing methods of the portland cement, the C_3A content, usually an important factor, is not always the principle factor correlated with resistance of concrete to sea water. In this sense, the recommended research by the Long-Time Study Advisory Committee that is being developed in San Pedro, Calif., will be a help.

Because it is not possible, for the time being, to give recommendations for portland cements exposed to sea water, the writer believes that experience should be a good guide: if concrete structures built with a given cement and exposed to sea water, show after many years of service good results, it will be possible to construct similar structures with the same cement and expect similar good results.

A possible reason why some high C_3A cements are more or less resistant to sulfate attack has been suggested by Parker, Bogue, and others[14] in that it is the C_3A that appears in the cement in crystallized stage that is harmful; but in some cases the C_3A is dissolved in glass, in this latter stage the C_3A is less harmful or harmless.

Thorvaldson[15] has suggested that the distribution of alumina between the three forms (the first, C_3A crystallized, the second, C_3A vitrificated, and third, should be alumina containing iron phase) is

determined by physical conditions during manufacture and possibly by the incidental presence of minor oxides.

By the way, Parker[14] has observed a relation between the calculated C_3A content and the sulfate attack only for cement low in glass (high in crystal).

J. H. P. Van Aart[14] presents another point of view and even though he believes in the possible importance of C_3A, he additionally suggests that curing conditions and the aggregates used have a major bearing on the subject. The suggestions are given and the way is open to investigation.

REFERENCES

7. Kleinlogel, Adolf, *Einflüse auf Beton und Stahlbeton*, Berlin, 1949 English and Spanish translations.

8. Shalon, R., and Raphael, M., "Influence of Sea Water on Corrosion of Reinforcement," ACI JOURNAL, V. 30, No. 12, June 1959 (*Proceedings* V. 55), pp. 1251-1268.

9. Fluss, Paul J., and Gorman, Sidney S., Discussion of Reference 6, ACI JOURNAL, V. 30, No. 6, Part 2, Dec. 1958 (*Proceedings* V. 54), pp. 1312-1322.

10. McMillan, F. R., *ACI Concrete Primer*, 1958.

11. McMillan, F. R., *Basic Principles of Concrete Making*, New York, 1929.

12. Hummel, Alfred, *Das Beton A B C Aquisgran*, Berlin, 1950. Spanish translation as *Prontuario del Hormigon*, Barcelona, Spain.

13. Hadley, Homer M., Discussion of Reference 6, ACI JOURNAL, V. 30, No. 6, Part 2, Dec. 1958 (*Proceedings* V. 54), pp. 1322-1324.

14. *Proceedings of the Third International Symposium on the Chemistry of Cement*, London, 1952.

15. Thorveldson, T., "Chemical Aspects of the Durability of Cement Products," *Proceedings of the Third International Symposium on the Chemistry of Cement*, pp. 436-466; Discussion pp. 466-484.

By HOMER M. HADLEY*

The writer has found this report of the 17-year performance of test piles in sea and supposedly fresh water an unusually interesting one. Here is the record of how the piles were made, the composition of their cements, how they were installed, what their exposures were, how they have performed—and in particular, how their performance relates the tricalcium-aluminate content of their cements.

As to their making, the limited cover of 1½ in. over their reinforcement is certainly to be regretted. Captain Blackman of Long Beach, Calif., a mighty advocate of 3 in. of cover over the reinforcement of plain, nonimpregnated concrete piles, has commended the statement appearing on p. 28 of the Portland Cement Association's booklet, *Concrete Piles*, issued Nov. 1939—"For unusually severe exposures, as in sea-water etc., a clear cover of 3 in. is recommended at least for the

*Member American Concrete Institute, Consulting Engineer, Seattle, Wash.

portion of the pile where such exposure is encountered." In this case there appears to have been a certain reluctance on the part of the physician to follow his own recommended remedy. As for the writer, he would seek to be at particular pains, with Mix 2 or better concrete, to have a full 2 in. of cover over lateral ties and would eliminate all cover with Mix 1 or Mix 3 concrete by eliminating those mixes in toto.

It is a question whether these specimens are properly to be described as piles. That name implies a specimen subjected to or capable of being subjected to, axial driving with a pile driver. With "1/4 in. round tie bars, spaced 3 ft apart" it is very little pile-driving they would have sustained. Probably "test-prisms" would describe them more fittingly.

It may appear a trifle unseemly to inquire, yet the 3 ft tie-bar spacing suggests the question: were there separate tie-bars for each group of four vertical bars or did a single tie bar serve as the lateral confinement of the entire cluster of eight vertical bars?

At the time the writer's observations on the behavior of concrete in sea water were printed—contemporaneously with the installation of these specimens—there was extant a strong feeling that the tricalcium-aluminate content of cement profoundly influenced the behavior of concrete in sea-water and that it were well, all things considered, to limit this content to 8 percent.[1] Again, quite recently, the same view has found expression with a proposed, and even more stringent limitation, tricalcium-aluminate content of cement to 5 percent.*

There appears to be no question that by what may be termed laboratory magnification, pronounced differences in their resistance to sea water can be shown to exist between cements of varying C_3A content. The great difficulty heretofore has been to correlate this laboratory experience with the behavior of structures in sea water. It just hasn't correlated. The detailed discussion by Messrs. Fluss and Gorman of the old San Francisco waterfront structures made with high C_3A cement, likewise that of H. E. Thomas respecting Pacific Gas and Electric Co. structures in San Francisco Bay, evoked by the Wakeman,[6] *et al.*, paper, "Use of Concrete in Marine Environments," emphasize the disparity between laboratory specimen and prototype. The Wakeman, et al, paper advocates a maximum limit of 5 percent C_3A. The writer, in his discussion of this paper, suggested that the authors examine and report on the present condition of the large concrete block base of the lighthouse on the end of the San Pedro breakwater built in 1910 with cement of 15 percent C_3A content.

Consequently, when the Salt Run "piles," made with Cement No. 18 whose C_3A content is 12.2 percent, deteriorate, that is "news"; here

*Reference 6, p. 841.

TABLE A

Cement No.	C_3A, percent
18	12.2
12	11.2
12T	11.1
11	10.9
15	9.9
31	9.6
33	9.2
13	8.9
14	7.1
16T	6.8
16	6.4
22	5.4
21T	5.4
21	5.1
43	4.8
24	4.4
43A	3.9
25	3.8
41	3.4
42	2.7
51	2.2
23	2.1

is deterioration in conjunction with 12.2 percent tricalcium aluminate cement, and all three piles affected, regardless of mix!

While Cement No. 18 does indeed have 12.2 percent C_3A content, there are other cements also having a fairly high C_3A content. This is shown in Table A which is tabulated on the basis of their descending values.

Arranged in order as shown in Table A, it can readily be seen that eight of the 22 cements would be rejected on the Stanton 8 percent C_3A basis, and six more of them on the Wakeman,[6] *et al.*, 5 percent C_3A basis. This leaves eight in the category of the more rigorous standard.

The next question and the really important one is: How do these criteria work out? Do they give any consistent and reliable indication of what may be expected from concrete made in accordance with them or don't they?

Confining our examination to the Mix 2 specimens, which are the only ones fit for sea water installation, at Cape Cod Canal the specimens made with Cements No. 16, No. 22, No. 24, and No. 31 have the poorest ratings and have C_3A percentages respectively of 6.4, 5.4, 4.4, and 9.6. At Saugerties the specimens are all much alike. That made with Cement No. 33, 9.2 percent C_3A, is somewhat worse than the others but as Table A readily shows there are six cements with higher C_3A content that behaved better than it did, as well as 15 with lesser percentages. At Salt Run three Mix 2 concretes as well as the concrete made with Cement No. 18 misbehaved, these others by cracking over the reinforcing bars. These were made with Cements No. 14, No. 41, and No. 24 with C_3A percentages respectively of 7.1, 3.4, and 4.4. At Corona del Mar, no Mix 2 specimens developed trouble despite the presence of Cements No. 11, and No. 18, each having a C_3A content of, respectively, 10.9 and 12.2 percent. Also, Cement No. 34 was used but the composition was not recorded in Table 1 in Mr. Tyler's paper.

Returning to the Salt Run specimens made with Cement No. 18 which developed "severe softening and deterioration" although none of the other specimens made with cements having a C_3A content greater than 8.9 percent have shown any weakness. How is this explained? In the course of a number of years of observations along the Pacific Coast, only one case of softening of concrete ever came to the writer's attention: that of the piling of the Ford Plant at Long

Beach. That trouble was clearly and definitely the use of feldspathic sand which by itself and without the presence of cement softened and disintegrated in sea water. The Long Beach trouble affected the entire pile section.

At Salt Run the trouble appears to be of an utterly unique character: deterioration confined to 2 or 3 in. of concrete adjacent to corners. That leaves 6 to 8 in. of sound unaffected concrete on each face of the specimen. As for the corner cracking, what is no more than a possibility is that these piles encountered difficult penetration and that the pile-driver foreman instead of merely letting his hammer weigh down the pile, hit them a few blows and the vertical bars bowed outward between the tie bars, "3 ft on centers." To the writer, however, this possibility appears about as significant as that the 12.2 percent C_3A content of the cement is responsible for the trouble. If it is, why is the concrete between the corners unaffected?

By R. F. STRATFULL*

The writer is aware of the fact that the original intent of this test was the evaluation of the relative performance of cements. That the corrosion of the steel has in some cases overshadowed the test of the performance of the cement is an unforeseen but fortunate occurrence. It is obvious that the corrosion of the steel should be considered in the over-all performance of a concrete structure exposed to an aggressive environment.

In reviewing the data given in the paper, further analysis reveals trends which could be considered indicative of the results that might be obtained in a more detailed investigation.

At St. Augustine and at Corona Del Mar, it appears that the presence of a crack in a concrete pile is a function of concrete quality, insofar as it indicates that sufficient sea salts have penetrated the concrete and resulted in the corrosion of the steel. Therefore, the absence or presence of a crack is considered an indirect but relative measure of concrete quality. The results of comparing the relative durability of concrete at these locations by means of the percentage of cracked piles are shown on Table B and C.

It appears that the progress of the corrosion of the steel, which is inferred from the length of a crack, is an additional variable that is controlled by concrete mass, moisture, and salt content, as well as by "concrete quality" but is not introduced into the computations because of absence of data showing the rate of growth of the cracks.

At Cape Cod Canal and Saugerties, the relative performance of the concrete was compared by evaluating the percentage of the piles in

*Corrosion Engineer, California Division of Highways, Materials and Research Department, Sacramento, Calif.

TABLE B—COMPARISON OF CONCRETE MIX BY PERCENT* DETERIORATION

	W/C	W/C — air entrained	St. Augustine		Cape Cod		Saugerties		Corona Del Mar, percent
			Non-air-entrained, percent	Air-entrained, percent	Non-air-entrained, percent	Air-entrained, percent	Non-air-entrained, percent	Air-entrained, percent	
Mix 1†	0.62	0.60	89	33	89	33	53	0	100
Mix 3†	0.53	0.51	47	0	56	33	16	33	86
Mix 2†	0.44	0.42	16	0	21	0	5	0	0

*Deterioration: St. Augustine, percent piles that are cracked; Cape Cod, percent piles that have rating greater than 3; Saugerties, percent piles that have rating greater than 2; Corona Del Mar, percent piles that are cracked.
†Mix 1, 5 sack, 2 in. slump; Mix 2, 7 sack, 2 in. slump; and Mix 3, 7 sack, 8 in. slump.

excess of an observed condition rating. The results of evaluating the concrete performance at these test sites by this method of analysis are also shown on Tables B and C.

The computations shown on Table C do not include Mix 1 (5 sack, 2 in. slump) at St. Augustine and Cape Cod Canal. The reason is that the majority of the piles made with Mix 1 at these locations have generally equalled or exceeded the comparative criteria or end point at some time in the past. As a result, there is no way of determining a comparative rate of deterioration of the concrete by this method of analysis.

In Table C the influence of the Types III and V cements on the deterioration of the piles was not analyzed as it is considered that there was an insufficient number of piles of these cement types for a direct mathematical comparison. Also, the cements used at Corona Del Mar were not compared in Table C because of inadequate data.

From a review of the test results, the following appear to summarize the performance of the concrete piles:

1. Type IV cement is superior to Types I and II cements in resisting the cracking of concrete from corrosion and from the other forms of concrete attack found at Cape Cod Canal and St. Augustine. Type II cement is generally superior to Type I in these same locations.

2. Without air entrainment, the concrete with the lower water-cement ratio is superior in all test locations.

3. In all locations, air entrainment appears to be of greater benefit than would be inferred solely on the basis of the small reduction in the water-cement ratio that accompanies the use of air entrainment.

4. Air-entrained concrete is superior to non-air-entrained concrete in resisting cracking of concrete from corrosion and also from the other forms of concrete attack found at Cape Cod Canal and Saugerties.

TABLE C—COMPARISON OF CEMENTS BY PERCENT* DETERIORATION

Type of cement, percent	St. Augustine, percent	Cape Cod, percent	Saugerties, percent
I	36	50	24
II	30	40	20
IV	25	0	0
All cements	34	35	17
Air entrainment	0	17	11

*Deterioration: St. Augustine, percent cracked for Mixes 2 and 3; Cape Cod, percent rated greater than 3 for Mixes 2 and 3; and Saugerties, percent rated greater than 2 for all mixes.

5. That a greater depth of concrete cover would have prevented corrosion in all of the piles is a matter of conjecture. However, it is reasonable to assume that a reduced amount of concrete covering the steel would have resulted in a higher percentage of concrete cracking at an earlier date.

6. It may be conjecture, but it appears that the minimum depth of concrete cover that is required to prevent the corrosion of the steel is a variable that depends upon the type of cement, the water-cement ratio, air entrainment, aggregates, and the time requirements for corrosion prevention.

7. In some cases the cracking of concrete due to the corrosion of the embedded steel was a greater problem than the concrete deterioration. Therefore, for reinforced concrete exposed to aggressive environments, the probability of corrosion of the steel should be considered in the over-all design of such structures as well as the deterioration of a concrete itself from exposure to the environment.

8. It appears the concrete that is more susceptible to attack by the environment is also the concrete that will more rapidly permit the corrosion of the steel. It does not necessarily follow that durable concrete will not be affected by the corrosion of embedded steel as the depth of the concrete cover is of prime importance.

Although this analysis of the results of the exposure of the piling at various locations is not conclusive, it appears that the results of this test support the ACI Standard "Recommended Practice for Selecting Proportions for Concrete (ACI 613-54)" in more aspects than the recommended water-cement ratio for concrete exposed to sea water. For instance, the test results appear to confirm the ACI Recommendations for the use of type of cement and air entrainment, as well as the water-cement ratio.

It is obvious that this test was not sufficiently broad to determine the relative performance of the Type V cement. However, the test indicated that concretes made with Types II and IV cements are more resistant to deterioration than those made with Type I cement. By observation and inference of the test results, it appears that concrete made with Type V cement should be more resistant to deterioration than Type I cement. Also, air entrainment increased the durability of concrete.

It appears that it would be of benefit to investigate the effect of various depths of concrete cover over steel and also aggregate variables,

such as size, shape, and type on the resistance of concrete to deterioration from weathering and corrosion. For instance, on Table 3 of ACI Standard 613-54, it is shown that the smaller the maximum size aggregates the greater the quantity of entrapped air in concrete which conceivably could have an effect on durability.

By LEWIS H. TUTHILL*

Not long ago an engineer asked, "What kind of concrete *will* prevent corrosion of its reinforcing in shore environments, even with ample cover?"

In confirming the advice in Table 4 of ACI Standard "Recommended Practice for Selecting Proportions for Concrete (ACI 613-54)," which states a water-cement ratio of from 4.5 to 5.0 gal. per sack of cement should be used for such concrete, Mr. Tyler's capable and revealing report of this phase of the Long-Time Study has gone a long way in giving us a surer answer to this question.

It is a common question and one that must be answered each time a shore area structure is built of concrete. Unfortunately to date, too often the answer or the field execution of it, has fallen short of providing the necessary protection.

From this phase of the Long-Time Study we now realize that these low *W/C* should not be confined to marine structures in a severe climate. The variable pentrations of salt water which set up galvanic corrosion cells can be just as destructive to reinforcing in a mild climate as in a severe one, perhaps more so because these salt solutions are not immobilized part of the time by freezing.

Except to make it quite evident that 1½ in. of cover is often inadequate, even for the lowest *W/C* mixes and particularly in freezing climates, the study did not provide enough data to enable a person to determine with confidence what practical depth of cover would provide long-range security from corroded reinforcement. We might speculate that in a saline atmosphere, security of cover might require a thickness of at least 2½ in. and at least twice the dimension of the largest aggregate.

With these considerations as to thickness of cover, we might further speculate that the answer to the question, "What is a protective concrete mix?", might well include the following requirements: (1) a maximum *W/C* of 4½ gal. per sack of cement or 0.40 by weight; (2) a rigid slump requirement in which all concrete having a slump over 3 in. is rejected; (3) air entrainment of 4 to 6 percent; (4) a cement that contains less than 4 percent C_3A, less than 12 percent total

*Member American Concrete Institute, Concrete Engineer, California Department of Water Resources, Sacramento, Calif.

aluminates, less than 50 percent C_3S, and less than 0.60 percent of alkalies equivalent to Na_2O; (5) a good pozzolan for about 15 percent by weight of the cement. Pozzolan has long had a favorable record in marine concrete and it would increase protection against a reactive aggregate. Moreover, it would reduce permeability and decrease stratification from bleeding. (6) a sound aggregate well graded to about 1 in. maximum size; (7) no aggregate or admixture that contains chlorides; (8) thorough water curing;

These items would add a few dollars to the cost of each cubic yard of concrete, but it would seem from the support given some of them by the Long-Time Study reported by Mr. Tyler, that such a premium would be well spent. Quite possibily this would make the difference between long-range serviceability and expensive repair or replacement of corroded reinforcing after a few years.

By CARROL M. WAKEMAN*

Users of cement owe a debt of gratitude to the Portland Cement Association for their part in instigating the Long-Time Study of Cement Performance in Concrete. Those who employ concrete for marine construction have particular reason to be grateful for this valuable research program.

Chapter 12 — "Concrete Exposed to Sea Water and Fresh Water" — has developed a number of interesting and useful facts and has, as Mr. Tyler pointed out, indicated fertile fields for future research.

Among the important lessons we have learned from the study is the value of an adequate cover of concrete over the reinforcing steel. Exactly how much cover is required depends on a number of variables, including workmanship, which conceivably could negate all other beneficial precautions.

Present day practice has established 3 in. as the minimum thickness of concrete over reinforcing steel. A much thinner cover is often desirable and, in the case of precast units, is frequently a necessity in order to minimize the weight factor.

Several research groups have recently reopened the question of the feasibility of coating reinforcing steel with modern types of hard, durable coatings, such as the epoxies, as a corrosion protection measure. If successful, both from the standpoint of retarding corrosion and adequate bond strength, a 1-in. cover of concrete may be practical.

The second fact, highlighted by the long-time study which, of course, has been suspected for years, was the value of rich mixes with low water contents.

---

*Member American Concrete Institute, Testing Engineer, Los Angeles Harbor Department, Wilmington, Calif.

It was not a great surprise that more rusting was observed in the mild climatic locations than in the colder zones. Corrosion of steel, being an electro-chemical reaction, it would follow that where concrete temperatures are higher, the speed of chemical reactions would be accelerated.

The long-time study has made it clear that chemical effects of sea water on concrete structures can best be observed in mild climates, because crumbling of concrete surfaces in subfreezing locations mask chemical deterioration.

It was noted that Mr. Tyler very properly expressed the rate of deterioration of the concrete in the Cape Cod and Saugerties locations, and those of St. Augustine, Florida, and Corona del Mar, California, in two different ways. Obviously, the action of freezing and thawing had perhaps a major influence in the deterioration at the first of the locations, while St. Augustine and Corona del Mar piles were not subjected to physical changes associated with freezing temperatures. In the writer's experience, it is quite difficult to distinguish the effects of sulfate deterioration of concrete from that of freezing and thawing except by chemical or petrographic examinations. In both cases, disintegration starts with the loss of bond of the outer aggregate particles, which are progressively removed by erosion. There is, however, a third condition that can take place in some environments that also produces softening of the outer surface of the concrete. It usually occurs only in leaner concrete mixes or those which have a preponderance of surface voids. Reference is made to concrete partly immersed in water relatively high in dissolved solids, such as sea water. In this case, the rise and fall of the tide can entrap pockets of sea water in voids above the normal high tide line. Evaporation results in supersaturation and finally the solids crystallize, accompanied by considerable expansion. The expansion in turn exerts sufficient force to pry off leading edges of voids and to expose further voids for a recycling of the crystallization process. The effect, and indeed the cause, could be similar to the action of freezing and thawing. Capillary action (depending on prevailing winds) can transport water upward for some distance above the high tide line, where crystallization of the solids and crumbling of the concrete can readily take place. This seldom happens to smooth, non-porous concrete surfaces where the cement content was adequate.

I realize that my remarks pertain largely to the performance of the finished product, concrete, rather than a component part, cement, which is the topic of Chapter 12. This seems justified in view of the fact that it is not yet feasible to test the raw product alone.

AUTHOR'S CLOSURE

The author would like to thank the five discussers for their interesting comments and criticisms concerning the paper. The fact that agreement is not complete is not at all surprising; if all the facts about marine exposure were known, there would be no need for such papers—or discussions of them. If the paper has contributed pertinent information or has stimulated useful debate, the author feels justified in its presentation.

Mr. Jamie de las Casas P. and Mr. Homer M. Hadley raise some questions regarding the need for avoiding high C_3A cements for sea water construction, citing the excellent record of the San Francisco Bay structures and others reported to contain high C_3A cements. The author has been interested in the San Francisco Bay structures for a number of years and has some satisfaction in knowing that concrete with high C_3A cement may stand up well in sea water. As a matter of fact there are several cements of moderately high C_3A content in the exposures reported that have shown no effects of chemical attack by sea water after nearly 20 years. Nevertheless the performance of Cement 18 with the many other similar bits of evidence seems to point to a deleterious contribution by high percentages of C_3A.

The author would like to assure Mr. Hadley that the three piles at Salt Run containing Cement 18 were not accidentally hit with the pile hammer; the problem there was to keep the piles from sinking out of sight when the jetting operation got under way. Only the single tie bar, 30 in. spacing, was used, mainly to hold the longitudinal bars in position during casting. The omission of data on composition of Cement 34 noted by Mr. Hadley is regretted. The calculated compounds by weight were as follows:

C_3S—67.3; C_2S—7.7; C_3A—5.2; C_4AF—9.2

From chemical analyses, percentages by weight were:

SO_3—1.72; free CaO—2.27; MgO—2.50; Na_2O—0.29; K_2O—0.29

The author offers no comment on Mr. Hadley's concern about damage being confined to corners of the Cement 18 specimens except to observe that corners are affected first in almost any kind of damaging attack. It may be of interest that at this writing all three of the specimens show softening across the faces and two of the three show decreases in pulse velocity near low-tide level.

Mr. R. F. Stratfull points out the very great difficulty of keeping cracking of the piles separated from other aspects of their performance. With this the observers of the installations would be in hearty agreement. Mr. Stratfull's analyses should be helpful in interpreting the data in the paper. It might be mentioned here that an investigation is under

way to study the performance of prestressed and nonprestressed reinforced concrete in sea water with concrete covers between 1 and 3½ in. over the steel.

Messrs. C. M. Wakeman and L. H. Tuthill are in agreement with the main points of the paper with Mr. Tuthill going considerably farther in his proposals for concrete in sea water. A number of Mr. Tuthill's proposals are now being tested in the recent addition to the Long-Time Study at San Pedro, Calif.

Discussion of a paper by M. F. Kaplan:

Effects of Incomplete Consolidation on Compressive and Flexural Strength, Ultrasonic Pulse Velocity, and Dynamic Modulus of Elasticity of Concrete*

By M. SPINDEL, M. R. VINAYAKA, and AUTHOR

By M. SPINDEL†

The investigation conducted by the author and the results published in the paper are another important contribution showing the effect of a high percentage of voids in concrete. The author showed especially the bad results of incomplete consolidation. This is, of course, only one of the many causes of harmful voids in concrete, which have been dealt with for a long time by leading concrete engineers and partly referred to by the author.

The first formulas considering the voids in concrete with regard to compressive strength and also to waterproof concrete were given by R. Feret‡ and Talbot and Richart.

These formulas, well known to concrete engineers in research and practice, could give the reduction of compressive strength due to the percentage of voids in a similar way as shown by the author.

Owing to circumstances which the writer does not wish to discuss, the same neglect happened to the use of his "fourmatter parallelogram" first published in the 50th anniversary edition of *Tonindustriezeitung Berlin* in 1926 and then supplemented and read by the writer at various international congresses and meetings in Zurich, Amsterdam, Vienna, Washington, and London and published in leading European periodicals between 1927 and 1936.§

With the fourmatter parallelogram for concrete it was only necessary to know for a certain volume of fresh concrete, the percentages of the weights of cement, aggregates, and water and the specific gravities of

*ACI JOURNAL, V. 31, No. 9, Mar. 1960 (*Proceedings* V. 56), p. 853. Disc. 56-47 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 32, No. 3, Part 2, Sept. 1960 (*Proceedings* V. 56).

†Member American Concrete Institute, Research Engineer and Consultant, London, England.

‡This formula by Feret was discussed thoroughly in a paper, "Discussionsbericht No. 7" and *Beilage* to it, published by the Director of the EMPA Zurich, M. Ros, in December, 1925.

§Here are mentioned only *Beton u. Eisen*, 1928, 1931, and 1935; *Zement*, 1936; and a paper on large dams, Washington, D.C., 1936, referred to in various discussions by the writer and Professor Eitel in the ACI JOURNAL.

the solids, i.e., mainly of the aggregates, to read in the parallelogram the percentage of voids and at the same time also the compressive strength and density of the concrete in question. There must have been some reason for not making more use of this fourmatter parallelogram in research laboratories and on the job.

It seems that in the first place it was due to the fact that concrete makers were used to giving the different substances for concrete either by volume or by weight, but there were no means whatever available to measure the amount of voids in fresh concrete until air entrainment meters were developed. Further, the air in fresh concrete varies in quantity during mixing, placing, and even setting and is thus more difficult to handle and control than cement, aggregate, and water.

It must not be overlooked that there are two different types of voids, those one cannot avoid, being not only harmful to strengths but also useful in saving costs. We now know that small amounts of air are useful to concrete, making it more resistant to frost, etc., but that the amount of air and its size, shape, and distribution have to be controlled.

In these circumstances the writer would like to give a short review of what happened to concrete making with regard to voids since the beginning of this century, especially in Europe where the writer made some contributions to improving cement and concrete during his activities in the Bridge Office and head of the Material Research Station of State Railways.

When the writer making concrete for railway bridges, etc., there were already government standards issued in 1904, considering the harmful role of voids in structural concrete. To avoid such voids these standards prescribed that as much cement had to be added as was necessary to fill the voids of sand and that as much cement and sand was needed as was necessary to fill the voids of coarse aggregate.

Although this seemed then and also today a reasonable demand, the writer found it did not work satisfactorily then and it does not do so now. The reason is that when adding cement to sand the voids in sand are greatly increased, so that the cement is not sufficient to fill the voids in the sand and the mortar is not sufficient to fill the voids in the coarse aggregate with the result that an appreciable amount of voids is left even if the concrete is consolidated so extensively as to break a lot of pieces of the large aggregate.

In these circumstances, especially for high strength or waterproof concrete, special studies had to be made with regard to the grading of cement, sand, and coarse aggregate, including gap grading. These studies led toward good results with regard to strength and impermeability, the latter even in water tanks of reinforced concrete without any rendering. The main result of the theoretical calculations by the writer was published in *Tonindustriezeitung Berlin* in 1913, showing by

mathematical calculation that in densely compacted spheres of a certain size, spheres of a smaller size could still be in the voids between the spheres of the large size, if the diameter of the smaller size was approximately 1/5 the larger size and the small size spheres could get through the spaces between if their diameter was less than 1/7 the large size.

The above calculation either for continuous or gap grading is still valid to get concrete with the least volume of voids. But, of course, there are many other points to be considered to avoid an intolerable percentage of voids, such as the grading and the shape and surface of the aggregates, the proportions between cement, sand, coarse aggregate, and water, the method of consolidation, and whether and which type of admixtures are used, which may reduce the water-cement ratio, increase the workability, entrain air, etc.

The consolidation of the concrete during placing is just the last part of concrete designing and making although of high importance. It should be useful to have some more information about the grading of fine and coarse aggregate and to learn whether the percentage of voids was also measured microscopically and whether there was agreement between the calculated and the microscopically tested amount of voids.

On this occasion the write would like to stress again the point already made in his previous discussions in the ACI JOURNAL that for structural concrete of high quality it does not suffice to measure or know the percentage of voids in it, but that the size, shape, and distribution of air voids have to be examined and measured microscopically. This has been done by some investigators, but the method has still to be improved. If so, it will show much better the advantages to be obtained by adding suitable admixtures to concrete, especially with regard to workability, water-cement reduction, air entrainment, and strengths.

In the discussion of the 1958 ACI JOURNAL paper on air voids* the writer pointed out the advantages of testing concrete microscopically with special objectives for dark ground illumination. Since then, he also developed another method of testing the air voids in concrete by viewing the air voids on nearly even concrete pieces with a metallurgical microscope using it like the "surface finish tester." This is bright field illumination visible only on lines and giving good results by illuminating the voids in a special way.

By M. R. VINAYAKA†

The paper is interesting and the data will prove useful to concrete technologists. Tests in our laboratory showed as much as 40 to 50 percent drop in compressive strength for 4 to 6 percent voids, though the mixes

*Discussion of "Origin, Evolution, and Effects of the Air Void System in Concrete," ACI JOURNAL, V. 30, No. 12, June 1959 (*Proceedings* V. 55), pp. 1355-1356.

†Member American Concrete Institute, Research Officer, Research Division, Koyna Project, Koyna, India.

tested were equivalent in consistency to 5-in. slump. Also the within-test variation of the specimens was very high. Can the author enlighten as to the variation noticed between the strengths of companion specimens tested by him? Will the author also throw light on the data, if any, regarding incomplete consolidation versus permeability of concrete in different directions?

AUTHOR'S CLOSURE

The author is pleased to receive Mr. Spindel's discussion and considers it to be a useful addition to the paper. The percentage voids was determined gravimetrically, as stated in the paper. In reply to Mr. Vinayaka, the standard deviation in the compressive strength of three test specimens varied from approximately 250 psi for average strengths of 1000 psi (coefficient of variation = 25 percent) to 500 psi for average strengths of 4000 psi (coefficient of variation = 12½ percent). As mentioned in the paper these variations are larger than those usually obtained for fully consolidated specimens under laboratory conditions. Permeability tests were not done.

Discussion of a paper by Tung Au and Donald L. Baird:

Bearing Capacity of Concrete Blocks*

By ŬGUR ERSOY, NEIL M. HAWKINS, and AUTHORS

By ŬGUR ERSOY†

The authors should be congratulated for approaching such an important problem. Block tests to determine the bearing capacity of concrete will have numerous applications in engineering practice. The writer would like to point out the importance of the bearing capacity of concrete in tall buildings, which was not mentioned by the authors.

In such buildings 5000-7000-psi concrete is being used in columns supporting beams and slabs of 3000-psi concrete to reduce the size of the column to a minimum. In the near future higher concrete strengths might be used. This raises the question of the bearing capacity of slabs and beams under high compression, caused by these highly stressed columns.

To use the data presented by the authors properly, the writer believes that the following points should be clarified.

1. In some specimens large aggregate having a maximum size of ½ in. have been used. The writer feels that these specimens cannot give a realistic picture of the behavior of actual structures. He also believes that in the application of the authors' test results to practical problems, only the specimens with small aggregate should be considered.

2. The writer would emphasize the importance of the ratio of the block thickness to the side dimension of the loading plate (referred to as t/d), which seems to be neglected by the authors.

The authors have based their theories on the formation of an inverted pyramid, underneath the loading plate, at failure. This is true for high t/d ratios, but the formation of the inverted pyramid shall be prevented in specimens having small t/d ratios. Such specimens will fail by crushing of concrete underneath the loading plate.

*ACI JOURNAL, V. 31, No. 9, Mar. 1960 (*Proceedings* V. 56), p. 869. Disc. 56-48 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 32, No. 3, Part 2, Sept. 1960 (*Proceedings* V. 56).

†Junior Member American Concrete Institute, Assistant Professor, Civil Engineering Department, Middle East Technical University, Ankara, Turkey.

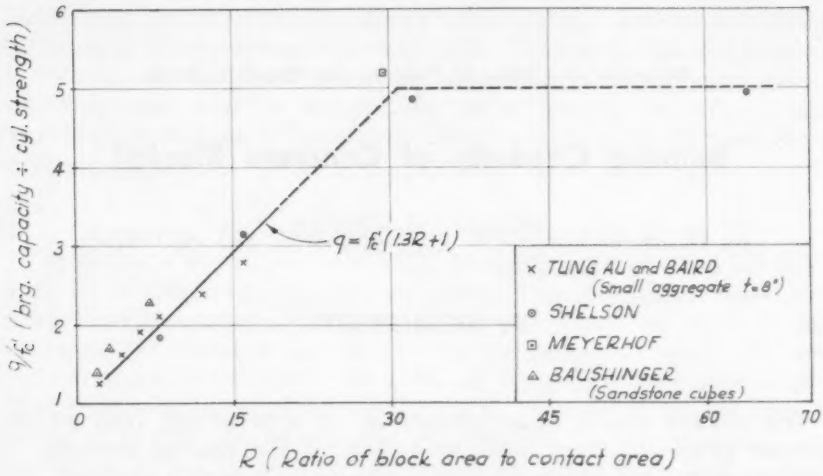


Fig. A—Test results on bearing capacity of concrete and stone

The writer feels, that for a fixed value of R, bearing capacity will be constant for blocks having high t/d decreases for small ratios (crushing failures).

3. The authors have pointed out that the bearing capacity of concrete blocks, subjected to localized load, would increase due to the confining pressure in the neighborhood of the contact area. In other words, bearing capacity would increase with the increasing value of R which was represented in Fig. 1. The writer fully agrees with this figure when R is small but believes that at a certain R value there will be adequate confining pressure in the neighborhood and increasing the ratio of the block area to the contact area from this point on will not affect the bearing capacity. In other words, the curve will assume a horizontal position at this point and q/f_c' will be constant from this point on.

To support this statement the test results of Shelson, Meyerhof, and the authors' (only specimens with small aggregate) have been plotted in Fig. A. This figure definitely indicates that the bearing capacity will be constant for high ratios of block area to the contact area. More tests are needed, especially to determine the horizontal portion of the curve.

4. All specimens have been tested under central loading. Tests made by the writer in 1957, when he was working with R. C. Reese (Consulting Engineer, Toledo, Ohio), indicated that the side and corner loadings would be more critical than the central loading. The writer believes that different kinds of loadings should be considered for future tests.

By NEIL M. HAWKINS*

The paper by Professor Au and Mr. Baird has clearly shown that the bearing capacity of concrete exceeds the unconfined compressive strength when the bearing area is less than the total area of concrete available. They are to be congratulated on having attempted a theoretical approach by proposing an equation as a rational basis for correlating the experimental constants to be used in the evaluation of the bearing capacity.

COMMENTS ON THE BASIC ASSUMPTIONS

To interpret their test results, Professor Au and Mr. Baird assumed that the downward penetration of the inverted pyramid prior to failure would cause horizontal pressures. The resultant of these pressures was assumed to produce combined tension and bending in the concrete block. Failure would then occur when the maximum tensile stress at the top of the block exceeded the tensile strength of the concrete.

The hypothesis of the formation of an inverted pyramid beneath the loaded area prior to failure is probably realistic. However, the assumption that the final failure by splitting is the result of combined tension and bending in the surrounding concrete block, does not seem reasonable.

The theoretical equation obtained from these assumptions

$$q/f_c' = \left(\frac{2s_o}{f_c'} + \frac{Kf_t}{f_c'}\right)\cot\alpha \qquad (1)$$

implies that q/f_c' is very sensitive to the depth of the block. It further implies that q/f_c' continues to increase without limit as the ratio of contact area to block area increases. Neither of these trends have been confirmed by tests.

ADDITIONAL TEST RESULTS

The results of five test series carried out over a period of several years at the University of Illinois are summarized in Table A. Each test series was cast from one concrete batch. Six 6 x 12-in. cylinders, for measurement of the unconfined compressive strength, and two 6 x 6 x 20-in. beams, for measurement of the tensile strength, were cast from the same batch. Test Series A, B, and C were cast using ⅜-in. maximum size aggregate. Test Series D and E were cast using ¾-in. maximum size aggregate. The concrete mixes had slumps varying from 1 in. to 3 in. To provide a smooth bearing surface, all specimens were turned on their sides. The specimens were loaded concentrically using square steel bearing plates ¾ in. thick.

*Member American Concrete Institute, Research Assistant in Civil Engineering, University of Illinois, Urbana, Ill.

TABLE A—TEST RESULTS

Test series	Type of specimen	Concrete strength, psi	Loading area, sq in.	Avg. least dimension, in.	R	Ultimate load, kips				Bearing capacity, psi	q/f_c'
						Specimen No.					
						1	2	3	Avg.		
A	6-in. cubes	3180	3.06	6.03	11.9	27.0	28.4	27.7	27.7	9,060	2.85
			4.14	6.02	8.8	37.5	35.1	32.5	35.0	8,450	2.66
			6.18	6.03	5.9	48.0	38.6	46.1	44.2	7,160	2.25
			9.09	6.00	4.0	52.0	58.0	54.0	54.7	6,020	1.89
			16.12	5.97	2.2	73.0	81.0	74.0	76.0	4,710	1.48
B	9-in. cubes	3780	6.18	8.98	13.0	74.0	74.0		74.0	11,970	3.16
			9.09	8.95	8.8	89.5	98.0		93.7	10,340	2.73
			16.12	8.93	4.9	116.0	136.0		126.0	7,820	2.07
C	6-in. cubes	7610	3.06	6.01	11.8	46.0	53.0	53.0	50.7	16,570	2.18
			4.14	6.00	8.7	59.5	67.5		63.5	15,340	2.02
			6.18	6.02	5.9	78.5	90.5	77.5	82.2	13,300	1.75
			9.09	6.07	4.1	110.5	97.0	100.5	102.6	11,290	1.48
			16.12	5.93	2.2	128.5	131.0	149.5	136.3	9,270	1.22
D	6-in. cubes	3840	4.00	6.00	9.0	46.8	44.5	49.0	46.8	11,700	3.05
			9.00	6.00	4.0	73.0	74.0	69.6	72.2	8,020	2.09
			36.00	6.00	1.0	134.0	131.5	132.0	132.5	3,680	0.96
E	6-in. cubes	5070	4.00	6.00	9.0	50.0	51.0	57.0	52.7	13,170	2.60
			9.00	6.00	4.0	70.0	76.0	72.5	72.8	8,090	1.59
			36.00	6.00	1.0	178.0	162.4		170.2	4,370	0.93

Table A lists the ultimate loads for each specimen tested. It contains the computed values of the average bearing pressure and the dimensionless quantity q/f_c' for each value of the ratio of block area to contact area for a given test series.

ANALYSIS

If the hypothesis of the formation of the inverted pyramid is accepted, it follows that the downward penetration of this pyramid must cause horizontal pressures. For any horizontal, cross-sectional, slice within the block above the level of the apex of this inverted pyramid, it may be assumed that these pressures will be distributed uniformly over the interface between the pyramid and the confining concrete. The situation is thus analogous to that of a thick walled square slice containing an axially symmetrical square cutout. This hollow square slice is subject to a uniform pressure over the internal surface.

To facilitate the development of a simple expression for the maximum uniform internal pressure that can be developed, the hollow square slice may be approximated by a thick-walled cylindrical slice. This is shown in Fig. B. The internal diameter d of the cylinder is obtained by inscribing a circle within the square cutout. The diameter d is equal to the length of the base of the pyramid at the level of the slice. The external diameter b of the cylinder is obtained by inscribing a circle within the bounds of the external sides of the surrounding concrete block. The diameter b is equal to the length of an external side of the block. If either the pyramid or the concrete block has a slightly rectangular instead of square cross section, the radii for the circles

of the cylindrical slice should be chosen so as to give the thinnest possible wall section. This method of inscription of the cylindrical slice ensures that the cylinder has the same wall thickness as the thinnest wall section of the square slice. A further simplification that may be made is to assume that the deformations of one horizontal slice do not interfere with the deformations of the neighboring slices. This assumption will only be true if the test specimen is of sufficient depth that the formation of the inverted pyramid beneath the loaded area is not restricted by the proximity of the lower contact surface. On the basis of this assumption expressions can be derived for the radial compressive stress σ_r and the tensile hoop stress σ_θ for any given slice of the thick walled cylinder.

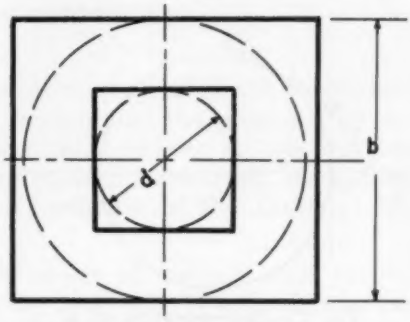


Fig. 8—Position of inscribed cylinder

At some value of the applied load the tensile strength of the concrete will be exceeded around the interface between the pyramid and the surrounding concrete block. Plastic action will follow. As the load is increased the width of the plastic zone in the concrete will increase. Failure will occur when the zone of plasticity has penetrated to the outer surface of the inscribed cylinder.

The internal pressure p_c at which failure will occur is given by[6]

$$p_c = 2s_o \log \left(\frac{b}{d}\right)$$

$$\text{or} \qquad p_c = s_o \log \left(\frac{b}{d}\right)^2$$

The thinnest wall section for all possible slices through the cross section of the pyramid is at the upper surface of the block.

The corresponding maximum internal pressure is

$$p_c = s_o \log R$$

For any given horizontal slice, the horizontal pressure p_c, exerted on the inner surface of the inscribed cylinder, is distributed over a perimeter of length πd; the horizontal pressure p_h exerted on the inner surface of the hollow square is distributed over a perimeter of length $4d$. It may be assumed that for any given slice the total force in a horizontal plane must be the same for both the cylinder and for the hollow square.

Equating the horizontal force on the cylindrical slice with that on the hollow square slice:

$$p_c \times \pi d = p_h \times 4d$$

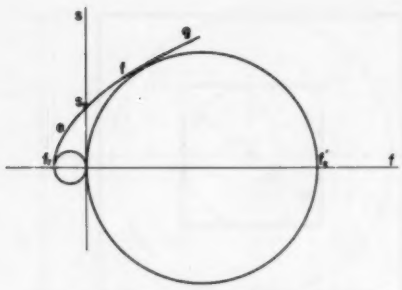


Fig. C—Determination of s_o

The maximum pressure on the hollow square slice that would correspond to the maximum internal pressure on the cylindrical slice is

$$p_h = \frac{\pi}{4} s_o \log R \quad \text{.....(2)}$$

This expression assumes that the deformations of the horizontal slice do not affect the deformations of the neighboring slices. Furthermore, it takes no account of the rate of variation with depth, of the total horizontal force on the pyramid of the actual block as compared to that on the cone for the inscribed cylinder. The expression takes no account of plastic action over a depth greater than that of the upper limiting slice so that Eq. (2) represents a lower bound to the total load that the block may carry. By contrast, the assumption that the concrete can strain sufficiently to develop complete plasticity throughout the width of the slice implies that Eq. (2) is an upper bound to the horizontal pressure that the slice may withstand. In view of these limitations, it is conceivable that Eq. (2) can be taken as a median for the ultimate horizontal pressure that will develop.

If the expression for p_h derived in accordance with Eq. (2) is used in the development of the expression for the ultimate bearing capacity, then Eq. (1) becomes

$$q/f_c' = \frac{s_o}{f_c'}\left[2 + \frac{\pi}{4}(\log_e R)\cot\alpha\right]\cot\alpha \quad \text{.....(3)}$$

The stress s_o can be determined from a Mohr's circle construction as shown in Fig. C. A parabolic envelope *egf* is drawn tangent to the tensile and compressive strength circles. The intersection of this parabola with the vertical axis gives the ultimate shear stress s_o.

It is of interest to note that when the loaded area becomes equal to the contact area, Eq. (3) reduces to

$$q/f_c' = 2\frac{s_o}{f_c'}\cot\alpha \quad \text{.....(4)}$$

If the test specimens are cubes, then for a given relation between cube strength and cylinder strength $(s_o/f_c')\cot\alpha$ must be a constant. Since $\cot\alpha$ will be determined from the test results for which the loaded area is less than the contact area, this limitation serves as another check on the assumed value of s_o.

CORRELATION OF TEST RESULTS

Effect of depth of test block

Consistent values for the bearing capacity of concrete will be obtained only if the depth of the test block is such that the proximity of the lower contact surface does not effect the formation of the inverted pyramid beneath the load area. If this condition is satisfied, the bearing capacity of the concrete will be independent of the depth of the specimen.

Fig. D illustrates the effect of the depth of the test block on the bearing capacity of the concrete. The ratio q/f_c' (bearing capacity/concrete strength) is the ordinate for any point and the value R (ratio of block area to contact area) is the abscissa. Results of four test series are plotted in Fig. D. The heights of the specimens vary from 6 to 9 in. Two different concrete strengths have been used. The squares represent test specimens with low concrete strengths and the circles represent test specimens with high concrete strengths. The hollow squares are for test Series A consisting of 6 in. deep cubes, and the solid squares are for test Series B consisting of 9 in. deep cubes. The hollow circles are for test Series C consisting of 6 in. deep cubes and the solid circles are for Messrs. Au and Baird's small aggregate series consisting of 8 in. deep cubes.

For a given value of R, there is a close correlation between the ratios of q/f_c' for similar concrete strengths. This shows that the bearing

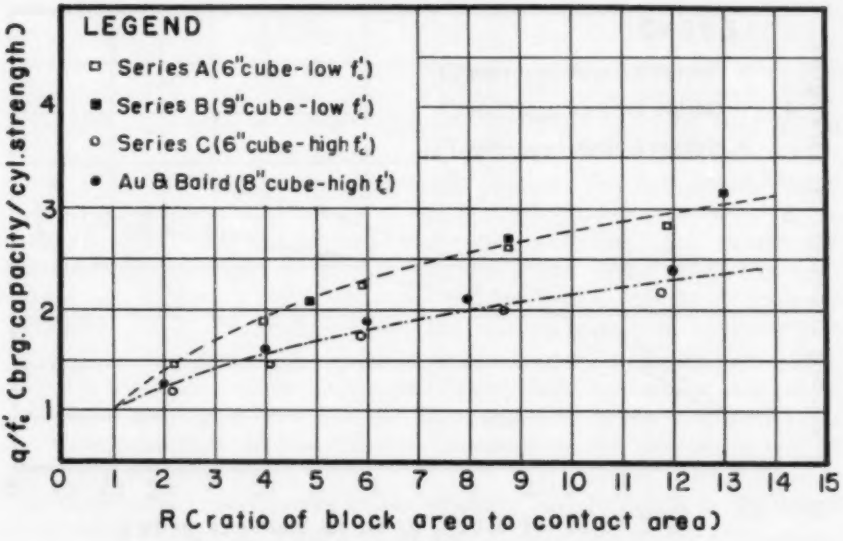


Fig. D—Effect of block depth and concrete strength

capacity of concrete is independent of the depth of the test block. Although the correlation is not as good for the high concrete strength as it is for the low concrete strengths, this discrepancy could be due to difference in the gradation and angularity of the aggregate used by Messrs. Au and Baird as opposed to the aggregate used at the University of Illinois.

Effect of concrete strength

The ratio of the tensile to the compressive strength diminishes for increasing compressive strengths and, as a result, the quantity s_o/f_c' will also decrease with increasing concrete strength. Accordingly, there should be a direct reduction of the same proportion as s_o/f_c' in the relative q/f_c' values for a given value of R, as the concrete compressive strength increases. This assumes that the corresponding alteration in cot α in accordance with the Eq. (4) is negligible.

In Fig. D, the test results represented by the circles are for high concrete strengths and the results represented by the squares are for low concrete strengths. For low concrete strengths the s_o/f_c' values are greater than for high concrete strengths. The corresponding q/f_c' values for any given value of R are higher for the low concrete strengths than for the high concrete strengths.

The ratio of tensile strength to compressive strength decreases as the maximum size of the aggregate increases. Differences in the angularity and gradation of the aggregate will however alter the angle of internal friction of the concrete. Any effect of a change in the maximum

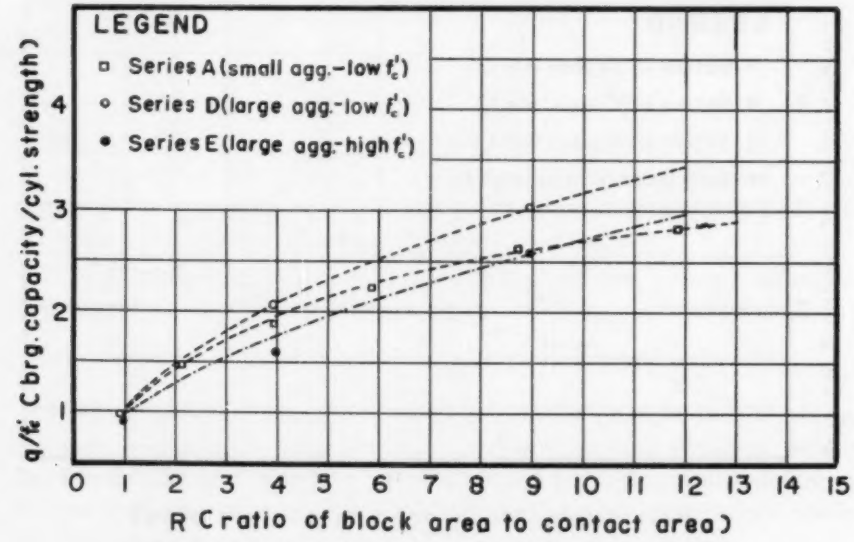

LEGEND
Series A (small agg.–low f'c)
Series D (large agg.–low f'c)
Series E (large agg.–high f'c)
q/f'c (brg. capacity/cyl. strength)
0 1 2 3 4
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
R (ratio of block area to contact area)

Fig. E—Effect of aggregate size and concrete strength

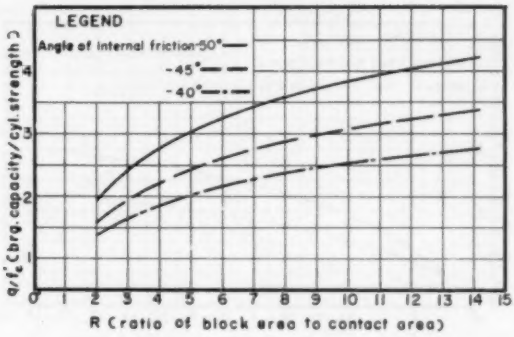

LEGEND
Angle of internal friction-50°
-45°
-40°
q/f'c (brg. capacity/cyl. strength)
0 1 2 3 4
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
R (ratio of block area to contact area)

Fig. F—Effect of an alteration in the angle of internal friction

aggregate size on the quantity s_n/f_c' may thus be effectively marred by a corresponding change in the angle of internal friction.

Effect of the angle of internal friction

According to Eq. (3), the bearing capacity of concrete varies directly with the angle of internal friction of the concrete. The value of the angle of internal friction depends, among other factors, on the density of the mix, the angularity and gradation of the aggregate, and the percentage of air voids. Values as low as 40 deg and as high as 50 deg may be expected.

For the materials used at the University of Illinois, the ¾-in. maximum size aggregate has a greater angularity than the ⅜-in. maximum size aggregate. Fig. E shows the effect of change in the maximum aggregate size on the q/f_c' versus R plot. The hollow squares are for test Series A with a maximum aggregate size of ⅜ in. The hollow circles are for test Series D with ¾-in. maximum size aggregate and low concrete strength and the solid circles are for test Series E, again with ¾-in. aggregate but with a higher concrete strength. Between any given values of R, the rate of increase in q/f_c' values for the test results using ¾-in. aggregate is much greater than for the test results using ⅜-in. aggregate.

Fig. E illustrates further two previous statements. The results for Series D and E again show that for any given value of R, the q/f_c' values are higher for low concrete strengths than for high concrete strengths. As R approaches unity, the results appear to confirm that the q/f_c' values for the specimens with the larger maximum size aggregate are less than values for the specimens with the smaller maximum size aggregate. This would be in accordance with the statement that the tensile strength of the concrete decreases as the maximum size of the aggregate increases.

A theoretical investigation of the effect of an alteration in the angle of internal friction is illustrated in Fig. F. The values of q/f_c', calculated from Eq. (3), are plotted for three different values of the angles

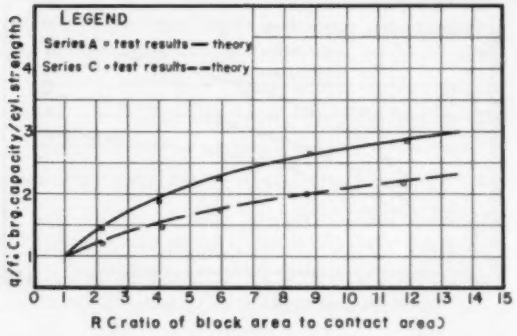

LEGEND
Series A ○ test results — theory
Series C ● test results - - theory
q/f'c (brg. capacity/cyl. strength)
R (ratio of block area to contact area)

Fig. G — Comparison between test results and theory, Series A and C

of internal friction. The lower curve is drawn for an angle of 40 deg, the center curve for an angle of 45 deg, and the upper curve for an angle of 50 deg. The quantity s_o/f_c' cot α has been assumed constant and equal to unity.

If Eq. (3) is assumed to be correct, this plot shows the desirability of well-compacted concrete in footings and under the anchorages of post-tensioned prestressed concrete beams, so that the angle of internal friction can be as high as possible.

Comparison of theory with test results

Fig. G shows the close correlation between the values of q/f_c', calculated in accordance with Eq. (3) and the measured values for test Series A and C. The test results for Series A are indicated by hollow squares and the results for Series C by hollow circles.

Fig. H shows the correlation between the theoretical and measured values for test Series D and the 8-in. cube, small aggregate size, for the test specimens of Messrs. Au and Baird. The test results for Series D are indicated by hollow circles and the results for Messrs. Au and Baird's tests by solid circles.

To simplify the calculations for both figures, the value of s_o was determined as outlined previously and the angle of internal friction

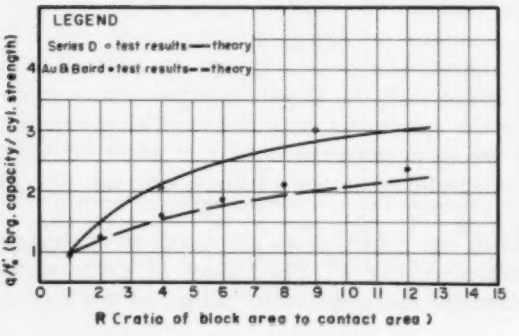

LEGEND
Series D ○ test results — theory
Au & Baird ● test results - - theory
q/f'c (brg. capacity/cyl. strength)
R (ratio of block area to contact area)

Fig. H — Comparison between test results and theory, Series D and Au-Baird

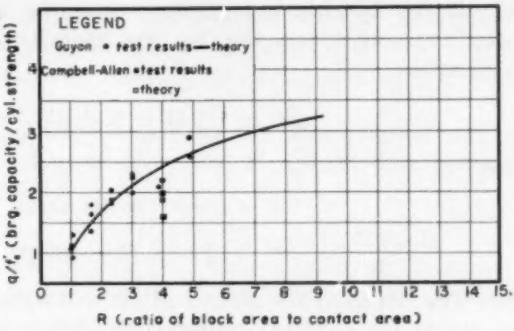

LEGEND
Guyon • test results—theory
Campbell-Allen •test results
atheory
q/f'c (brg. capacity/cyl. strength)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
R (ratio of block area to contact area)

Fig. I—Comparison between test results and theory, Guyon and Campbell-Allen

was assumed constant at 43 deg. Although the agreement is not as close in Fig. H as it is in Fig. G, the deviation is not unreasonable. As discussed previously, a higher value for the angle of internal friction should be used for Series D than for either Series A or C. Furthermore, the theoretical curve for Messrs. Au and Baird's series has to be based on an assumed value for the tensile strength of the concrete which could easily be in error.

Comparison with other test results

If Eq. (3) is adapted to apply to a cylindrical block, the theoretical results are in good agreement with those reported by Campbell-Allen[7] and Guyon.[8]

For a cylindrical test specimen Eq. (3) becomes

$$q/f_c' = \frac{g_o}{f_c'}[2 + (\log_e R) \cot \alpha] \cot \alpha \quad \ldots\ldots (5)$$

In Fig. I the solid circles represent test results reported by Guyon and the curve represents the theoretical result obtained in accordance with Eq. (5). On the same figure the two solid blocks represent test results reported by Campbell-Allen, and hollow block above each of these in turn represents the theoretical result calculated from Eq. (5).

Eq. (3) has not been extended to cover the other test results reported by Shelson[2] or Campbell-Allen[7] because of uncertainties as to the relation between the tensile and compressive strengths of the mortars that were used in these experiments.

It may be possible to extend the results outlined in this discussion to cover rectangular bearing blocks as well as square bearing blocks.

CONCLUSIONS

As has been pointedly shown by the authors in their original paper the bearing capacity of concrete is influenced by a multiple of variable factors.

From the additional results reported in this discussion it can be concluded that:

(1) If the bearing area is less than the total surface area of the loaded concrete block, the bearing capacity can be determined from the equation

$$q/f_c' = \frac{s_o}{f_c'} [2 + K (\log_e R) \cot \alpha] \cot \alpha$$

where

$K = 1$ for circular areas

$K = \frac{\pi}{4}$ for square areas

(2) The depth of the loaded block will have no effect on the bearing capacity of the concrete provided the block is of sufficient depth that the proximity of the lower contact surface does not restrict the formation of the inverted pyramid beneath the loaded area.

(3) The bearing capacity of the concrete will approach some limiting value as R increases indefinitely.

(4) Since the bearing capacity of concrete is dependent directly on the angle of internal friction it is desirable that the in situ concrete be as dense as possible, and that air voids be kept to a minimum.

ACKNOWLEDGMENTS

The results reported as test Series D and E were performed by John R. Ross and John P. Holmes under the direction of Prof. Clyde E. Kesler, Department of Theoretical and Applied Mechanics, University of Illinois.

REFERENCES

6. Seely, F. B., Smith, J. O., *Advanced Mechanics of Materials*, John Wiley and Sons, New York, 1959, pp. 295-340.

7. Campbell-Allen, D., Discussion of "Bearing Capacity of Concrete," ACI JOURNAL, V. 29, No. 12, June 1958 (*Proceedings* V. 54), pp. 1185-1187.

8. Guyon, Y., *Prestressed Concrete*, V. 1, John Wiley and Sons, New York, 1955, pp. 156-158.

AUTHORS' CLOSURE

The authors appreciate the comments and constructive criticisms by Messrs. Ersoy and Hawkins which have materially improved the value of the paper.

Since the bearing capacity of concrete is influenced by a multiple of variable factors, it was neither possible nor desirable to generalize the conclusion of the paper beyond the range of variables considered. The authors are greatly indebted to Mr. Hawkins for reporting the results of other tests conducted at the University of Illinois as well as providing an excellent interpretation of the test data heretofore avail-

able. While certain approximations are involved in the derivation of his equation, the assumptions introduced in simplifying the problem are reasonable. The conclusions reached by Mr. Hawkins are indeed convincing.

Practically all of the questions raised by Mr. Ersoy have been answered in the light of the discussion by Mr. Hawkins. It is necessary to add only a few remarks.

1. The bearing capacity of concrete is dependent directly on the angle of internal friction which may be affected by many factors. In making laboratory tests, it is not just a matter of scaling down the size of the aggregate of the test specimens.

2. While the bearing capacity is independent of the depth of the block so long as the depth is great enough to permit the development of an inverted pyramid under the bearing area, the bearing capacity of a shallow block has been found to be slightly higher. In Fig. 2 test results indicate that, for the same value of R, the bearing capacity is greater for the 4 in. deep blocks than for the 8-in. cubes.

3. The limiting value of the bearing capacity is defined by the equation suggested by Mr. Hawkins. Although no direct correlation between the theoretical values based on that equation and Shelson's experimental data for large values of R has been made, it seems that the theoretical expression represents a lower bound of the bearing capacity.

4. Tests of eccentrically located circular bearing areas on cylindrical blocks have been reported by Campbell-Allen,[7] but no attempt has been made by the authors to correlate his data with those based on centrally located square bearing areas.